Numerical Simulations of Steady and Oscillatory Flows around Multiple Cylinders

by

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This thesis is presented for the degree of
Doctor of Philosophy
of
The University of Western Australia

Civil Engineering
School of Civil, Environmental and Mining Engineering

May, 2014
DECLARATION FOR THESIS CONTAINING PUBLISHED WORK AND/OR WORK PREPARED FOR PUBLICATION

This thesis contains published work and/or work prepared for publication, which has been co-authored. The bibliographical details of the work and where it appears in the thesis are outlined below.

The estimated percentage contribution of the candidate is 60%.

TONG, F., CHENG, L. & ZHAO, M., 2014. Classification of wake flow patterns around four cylinders in a square arrangement in steady flow. Under review. (Chapter 3)
The estimated percentage contribution of the candidate is 60%.

The estimated percentage contribution of the candidate is 60%.

The estimated percentage contribution of the candidate is 60%.

TONG, F., CHENG, L. & AN, H., 2014. The scalability of OpenFOAM on two supercomputers for simulations on flow around a cylinder. Preparation for submission (Appendix)
The estimated percentage contribution of the candidate is 60%

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Signature ___________________________ Date ___________________________
Abstract

This study numerically examines the flow features around multiple-circular cylinders and hydrodynamic forces on the cylinders. The research is motivated by both practical engineering applications and understanding of fundamental flow characteristics. The thesis structure and major findings from this study are summarized below.

Steady uniform flow around two identical circular cylinders of various arrangements at a subcritical Reynolds number (Re) is investigated in Chapter 2. The pitch distance between the cylinders (P) and the cylinders’ alignment to the cross flow are found to be highly influential on the pressure distributions on the cylinders, the vortex shedding frequency (St), and the enstrophy of the system. The change in pressure distribution leads to a variety of characteristics of the forces on both cylinders, including negative drag force, attractive and repulsive lift forces, and the suppression or amplification of these forces compared to the those on an otherwise isolated cylinder. Two distinct values of St are identified in the staggered arrangement based on force analysis. Flow regimes around the two-cylinder system with the change of P and alignment angle are identified and classified. It is found that three-dimensionality of the flow in the gap region and in the shared wake is considerably weakened by cylinder interference in two of the flow regimes.

Chapter 3 is highlighted by a total of seven flow regimes induced by steady flow around a four-cylinder array in a square arrangement, which are identified and mapped on the plane of Reynolds number and pitch distance. Each of the seven flow regimes has distinguished flow features, resulting from the interactions among the shear layers, vortices shed around cylinders and Kármán vortex streets behind the them. Some of the flow features around the four cylinders, such as the single bluff-body vortex shedding, binary vortex shedding, in-phase vortex shedding, biased vortex shedding, and vortex co-shedding & anti-phase synchronization, share similarities with flow features exhibited by flow around two side-by-side cylinders and flow around two tandem cylinders. Some of the flow features, such as triple vortex streets and the in-phase
escape of vortices at the onset of vortex shedding from upstream cylinders found in the in-phase vortex shedding regime, are unique to the four cylinder system and have not been reported previously.

Three-dimensional (3-D) simulations are carried out in Chapter 4 to study the vortex shedding flow in the wake of four circular cylinders in a square configuration at one chosen pitch distance where four distinct flow regimes are identified. Physical mechanisms responsible for different flow regimes are proposed and discussed in details. Significant changes in the force coefficients, wake formation length and phase angle of the lift coefficients on the downstream cylinders are observed when the flow transits from one regime to another.

Sinusoidally oscillatory flow around four circular cylinders in the in-line square arrangement is modeled in Chapter 5 at oscillations with relatively low frequency and low amplitude. The primary aim is to investigate the influence of cylinder proximity on flow regimes. A captivating set of flow patterns is observed and identified, including six types of reflection symmetry to the axis of oscillation, two types of spatio-temporal symmetry and a series of symmetric breaking flow patterns. In general, at small gap distances, the four cylinders behave as a single porous body and therefore, the flow fields resemble to those around a single cylinder; while with the increase of gap distance, the individual behavior of each cylinder in the array starts to dominate the flow patterns and thus, the flow field shows a variety of symmetry states as a result of vortex interactions from each cylinder, but also is very prone to asymmetry. The force distributions present the similar features to those observed in flow fields.
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Acknowledgements

First of all, I would like to express my sincere gratitude to my supervisor, Winthrop Professor Liang Cheng, for his close supervision, persistent interest and incisive guidance on the research, and particularly for the time that he has generously devoted on paper revisions and on valuable meetings with myself. My debt to him is beyond measure for all the support that he has provided and I feel blessed having him in the path of the study.

I am extremely grateful to Dr. Ming Zhao from the University of Western Sydney for his insightful revisions and constructive comments on the papers involved in this thesis. My heartfelt thanks go to Prof. Tongming Zhou, Dr. Xiao-bo Chen from BV France and Dr. Hongwei An for their effort and enormous help towards the completion of this thesis.

Many thanks go to my fellow group members for their friendship and diverse help. I also would like to extend my special thanks to many staff and postgraduate student from CRE and COFS, former and present, who provided assistance on various occasions.

Acknowledgement is also due to the Australian Government and the University of Western Australia, who provided SIRF, UIS and PhD completion scholarships to my PhD study. This work was also heavily supported by staff and resources from iVEC through the use of Epic, Magnus and Cortex supercomputers.

I want to thank my parents, grandparents, and my siblings, for their love, care and understanding. I consider myself as very fortunate having them in my life.

Finally, I wish to express my thanks to my wife and best friend, Ms Wei Sun. Nothing is warmer than her accompany.
Thesis Organization and Candidate Contribution

In accordance with the University of Western Australia’s regulations regarding Research Higher Degrees, this thesis is presented as a series of papers that have been published, accepted for publication or submitted for publication but not yet accepted. The contributions of the candidate for the papers comprising Chapters 2 ~ Appendix A are hereby set forth.

Paper 1

This paper is presented in Chapter 2, first-authored by the candidate, co-authored by Winthrop Professor Liang Cheng and Dr. Ming Zhao, and submitted as


The candidate developed the program and carried out the simulations on steady uniform flow around two identical circular cylinders of various arrangements. Under the supervision of Winthrop Professor Liang Cheng and Dr. Ming Zhao, the candidate overviewed relevant literature, carried out parametrical studies, interpreted the results and wrote the paper.

Paper 2

This paper is presented in Chapter 3, first-authored by the candidate, co-authored by Winthrop Professor Liang Cheng and Dr. Ming Zhao, and submitted as

The candidate studied the flow regimes induced by steady flow around a four-cylinder array in a square arrangement with 2-D numerical simulations. Under the supervision of Winthrop Professor Liang Cheng and Dr. Ming Zhao, the candidate overviewed relevant literature, carried out parametrical studies, interpreted the results and wrote the paper.

**Paper 3**

This paper is presented in Chapter 4, first-authored by the candidate, co-authored by Winthrop Professor Liang Cheng and Dr. Ming Zhao, Prof. Tongming Zhou, Dr. Xiaobo Chen, and published as


The candidate studied the 3-D flow features around four circular cylinders in the square arrangement and investigated the forces on the cylinders. Under the supervision of Winthrop Professor Liang Cheng and Dr. Ming Zhao, Prof. Tongming Zhou, Dr. Xiaobo Chen, the candidate overviewed relevant literature, carried out parametrical studies, interpreted the results and wrote the paper.

**Paper 4**

This paper is presented in Chapter 5, first-authored by the candidate, co-authored by Winthrop Professor Liang Cheng and Dr. Ming Zhao, Dr. Hongwei An, and submitted as


The candidate studied the oscillatory flow around four circular cylinders and classified the flow regimes. Under the supervision of Winthrop Professor Liang Cheng and Dr. Ming Zhao, and Dr. Hongwei An, the candidate overviewed relevant literature, carried out parametrical studies, interpreted the results and wrote the paper.
Paper 5

This paper is presented in Appendix A, first-authored by the candidate, co-authored by Winthrop Professor Liang Cheng and Dr. Hongwei An, and is under preparation for submission as

- TONG, F., CHENG, LIANG & AN, H.,, 2014. The scalability of OpenFOAM on two supercomputers for simulations on flow around a cylinder. Preparation for submission

The candidate investigated the relationship between speed of CFD calculations and the number of CPUs employed. Under the supervision of Winthrop Professor Liang Cheng and Dr. Hongwei An, the candidate overviewed relevant literature, carried out parametrical studies, interpreted the results and wrote the paper.

Signature:  ______________________

Feifei Tong

May 2014.
Publications Arising from This Thesis

Journal papers


Conferences papers

configuration. 6th International Conference on Asian and Pacific Coasts, Hong Kong, pp. 1322-1328.
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</table>
Chapter 1

Introduction

1.1 Background

In fluid mechanics, vortex shedding describes the periodic detachment of swirling fluid flow in the wake of cylindrical structures. The occurrence and the frequency (when it does happen) of vortex shedding are dependent on the velocity and viscosity of the fluid as well as the size of the structure. This observation is named after Osborne Reynolds (1842–1912) as Reynolds number, written as \( Re = \frac{UD}{\nu} \), where \( D \) is the diameter the cylinder, \( U \) is the free stream velocity and \( \nu \) is the kinematic viscosity of the fluid.

1.1.1 Steady flow around a single cylinder

Vortex shedding from a bluff body in steady flow has been studied extensively in the past decades due to its importance in engineering applications, such as to prevent vortex-induced vibrations, and its richness in fundamental fluid mechanics, such as when vortex sheds from a structure and how the flow behaves in the wake. Much knowledge has been achieved on flow past a single cylinder through these studies and comprehensive reviews can be found in Bearman (1984), Williamson (1996b), Sumer and Fredsøe (1997) and Williamson and Govardhan (2004).

Flow in the wake of a circular cylinder can be categorized into several regimes with the change of Reynolds numbers (Williamson, 1996b, Sumer and Fredsøe, 1997). The
classification is based on observations in the flow features, as well as measurements on base suction pressure coefficient, mean drag force coefficient and the vortex shedding frequency, some of which are listed in Figure 1-1 from several published sources.

![Figure 1-1 Classification on steady flow regimes around a single circular cylinder as illustrated by the changes of base pressure, mean drag force coefficient and Strouhal number; (a), base pressure, figure reproduced from Williamson (1996b); —, Henderson (1995); ○, Williamson and Roshko (1990); data larger than $10^3$ from various sources, refer to Williamson (1996b); (b) mean drag force, figure reproduced from Schlichting and Gersten (2003); (c) Strouhal number, data smaller than $4 \times 10^2$ from Williamson (1988) and larger than $2 \times 10^4$ from Schewe (1983); the dash and dotted lines are two sources from Achenbach and Heinecke (1981).]

The classified regimes from Williamson (1996b) is shown in Table 1-1 and their
The nomenclature is adopted in the discussion. Totally eight flow regimes are defined and
the approximate ranges are shown in Figure 1-1 (a) and the Reynolds number values are
listed in Table 1-1.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Range</th>
<th>Re</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar steady</td>
<td>Up to A</td>
<td>&lt; 49</td>
</tr>
<tr>
<td>Laminar vortex shedding</td>
<td>A-B</td>
<td>49 ~ 140-194</td>
</tr>
<tr>
<td>3-D Wake-transition</td>
<td>B-C</td>
<td>190 ~ 260</td>
</tr>
<tr>
<td>Increasing disorder in three dimensionality</td>
<td>C-D</td>
<td>260 ~ 10^3</td>
</tr>
<tr>
<td>Shear-layer transition</td>
<td>D-E</td>
<td>10^3 ~ 2 x 10^7</td>
</tr>
<tr>
<td>Asymmetric reattachment (Critical transition)</td>
<td>E-G</td>
<td>2 x 10^7 ~ 4 x 10^5</td>
</tr>
<tr>
<td>Symmetric reattachment (Supercritical)</td>
<td>G-H</td>
<td>4 x 10^5 ~ 1 x 10^6</td>
</tr>
<tr>
<td>Boundary-layer transition (Post-critical regime)</td>
<td>H-J</td>
<td>1 x 10^6 ~ 1 x 10^7</td>
</tr>
</tbody>
</table>

Table 1-1 Classifications on flow feature around a circular cylinder in cross flow,
based on Williamson (1996b).

* Ranges of Re are estimated from Figure 1-1

No vortex shedding is detected at the Re lower than 49 (laminar steady regime),
where the wake is featured with two steady circulation zones (Coutanceau and Bouard,
1977, Henderson, 1995). The mean drag force decreases monotonically with the
increase of Re at this regime.

When Reynolds number is small (Re = 49 ~ 140-194), the vortex flow is two-
dimensional (2-D, laminar vortex shedding regime). The shed vortices move with the
fluid in the downstream, forming a classical configuration of vortex street in the cross-
section, which is usually referred to as Kármán vortex street (after Theodore von

As Reynolds number is increased to around 180 ~ 194, secondary instability in the
wake flow field appears. The three-dimensional instability, which is characterized by
the inception of streamwise vortex loops with a spanwise wavelength of 3 ~ 4 times of
the cylinder diameter and a sudden decrease in Strouhal frequency, is referred to be
mode A (Williamson, 1988, Williamson, 1996a). The spanwise wavelength of the
streamwise vortex pairs decreases from around 3 ~ 4D to approximately 1D as
Reynolds number is further increased to 230 ~ 250, where the mode A instability
gradually gives way to the so called mode B instability. The wake fulfills the transition
from 2-D to 3-D in mode B (Williamson, 1996b).

With the further increase in Reynolds number, the flow around the structure is
sequentially associated with disorderly fine scaled vortices in the wake (Prasad and
Williamson, 1997, Williamson, 1996a), and 2-D to 3-D turbulence transition in the shear layers (Norberg, 1994), where the magnitude of base pressure experiences a steady rise.

The critical transition regime is featured with drastic decrease in base pressure and the drag force [also known as drag crisis, Sumer and Fredsøe (1997)], but extreme increase in vortex shedding frequency (Schewe, 1983), in a relatively small range of Reynolds number. The flow in this regime is also found to be asymmetric to the free stream direction and bi-stable, leading to rather large lift forces.

After the supercritical regime, characterized by greater base pressure and larger drag force than that in any other regimes, with the increase in Reynolds number the turbulence transition moves further upstream to the boundary layer of the cylinder. Therefore, the flow in the wake of a circular cylinder experiences a series of instabilities from the far field to the near field with the increase of Reynolds number.

1.1.2 Steady flow around multiple cylinders

Steady flow around a cylinder is substantially affected when it is placed in the proximity of another one. The vortex shedding pattern, the force coefficient and the pressure distribution for a cylinder array are greatly influenced by the spacing among the cylinders and their arrangements, in addition to the Reynolds number. The simplest case of a group of cylinders in close proximities is two identical circular cylinders with various arrangements, which have been the interest of many studies in the recent decades (Zdravkovich, 1987, Xu and Zhou, 2004, Sumner, 2010). Thus the pitch distance \( P \) (distance between the two cylinder centers) and alignment angle \( \alpha \) (the angle between center-to-center line and the direction of the cross flow) are introduced as additional parameters that affect the flow.

Zdravkovich (1977, 1987) classified the flow field around two cylinders into two basic types of interference based on the location of the downstream cylinder with respect to the upstream one, i.e., the wake interference and the proximity interference. The wake interference is obvious when the two cylinders are in tandem \( (\alpha = 0^\circ) \), where one cylinder is completely submerged in the wake of the other. The proximity interference occurs when the two cylinders are close to one another but neither is in the wake of the other, as in the side-by-side configuration \( (\alpha = 90^\circ) \). Both wake and proximity interferences present in flow around two cylinders in a staggered
arrangement. A rich spectrum of flow features is generally associated with these interferences..

Igarashi (1981, 1984) identified eight different flow regimes for flow past two tandem cylinders, which were later narrowed down into three (Xu and Zhou, 2004, Zdravkovich, 1987, Sumner, 2010), namely, (i) extended-body regime at small pitch, where the two cylinders behave as a single bluff-body; (ii) reattachment regime at intermediate pitch, where the separated shear layers from the upstream cylinder reattach to the downstream cylinder; and (iii) co-shedding regime at large pitch, where Kármán vortices are shed from both cylinders. However, the overlaps in the pitch ranges of the flow regimes are observed in difference studies, likely due to the differences in Reynolds number in the studies (Xu and Zhou, 2004, Zdravkovich, 1987, Sumner, 2010).

The flow around two side-by-side cylinders have also been extensively studied, and were classified into three regimes by Sumner (2010). They are (i) single-bluff-body at small pitch, where the two cylinders are close enough to act as a single structure; (ii) biased flow regime at intermediate pitch, where the gap flow biased towards one of the two cylinders, resulting in an asymmetric flow field; and (iii) parallel vortex streets at large pitch, where the flow field is symmetric with one Kármán vortex street behind each cylinder. Again, due to the flow’s sensitivity to Re, pitch ratio overlaps between two adjacent flow regimes.

Flow around two cylinders in the stagger arrangement is also commonly encountered in practical applications. Most experimental studies about flow past two staggered cylinders were carried out at high Reynolds numbers in the subcritical regime. Classifications of flow regimes around two staggered cylinders have been discussed in several studies. By measuring pressure distributions and carrying out flow visualization, Gu and Sun (1999) recognized three flow patterns for \( P/D = 1.1 \sim 3.5 \) and \( Re = 2.2 \times 10^5 \sim 3.3 \times 10^5 \), which are wake interference, shear layer interference and neighborhood interference. On the basis of flow visualization images and instantaneous PIV data at Re = 850~1900, Sumner et al. (2000) identified nine different flow patterns, including many flow field interactions, such as shear layer reattachment and separation, vortex pairing, synchronization and impingement. The studies by Hu and Zhou (2008a, 2008b) were focused on the evolution of flow structures, Strouhal number in the wake, and the heat and momentum transfer in the flow in the wake of two staggered cylinders. The
wake flow fields at a Reynolds number of \( Re = 7000 \) were classified into four regimes based on flow visualizations and Strouhal numbers, including two single-wake regimes and two twin-wake regimes. More flow regimes were identified at low \( Re \) (Sumner et al., 2000) than that at high \( Re \) (Gu and Sun, 1999, Hu and Zhou, 2008a). It is observed in those studies that the flow behavior is heavily dependent on the arrangement of the cylinders. The experimental studies on flow around two staggered cylinders, have been comprehensively reviewed by Sumner (2010). The flow patterns around two staggered cylinders can be broadly classified into three regimes, which are named as single-bluff-body flow regimes, small-incidence-angle flow regimes, and large-incidence-angle flow (Sumner, 2010). The flow regimes around two tandem and two side-by-side cylinders are the fundamental elements of the flow patterns identified for two staggered cylinders.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>( Re )</th>
<th>( P ) and ( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sayers (1988, 1990)</td>
<td>Experimental</td>
<td>( 3 \times 10^4 )</td>
<td>( 1.1 \leq P \leq 5, 0^\circ \leq \alpha \leq 45^\circ )</td>
</tr>
<tr>
<td>Lam and Lo (1992)</td>
<td>Experimental</td>
<td>2100</td>
<td>( 1.28 \leq P \leq 6, 0^\circ \leq \alpha \leq 45^\circ )</td>
</tr>
<tr>
<td>Lam and Fang (1995)</td>
<td>Experimental</td>
<td>( 1.28 \times 10^4 )</td>
<td>( 1.26 \leq P \leq 5.80, 0^\circ \leq \alpha \leq 45^\circ )</td>
</tr>
<tr>
<td>Farrant et al. (2000)</td>
<td>Numerical</td>
<td>200</td>
<td>( P = 3&amp;5, \alpha = 0^\circ&amp;45^\circ )</td>
</tr>
<tr>
<td>Lam et al. (2003a)</td>
<td>Experimental</td>
<td>200</td>
<td>( P = 4, 0^\circ \leq \alpha \leq 45^\circ )</td>
</tr>
<tr>
<td>Lam et al. (2003b)</td>
<td>Experimental</td>
<td>200&amp;800</td>
<td>( 1.69 \leq P \leq 3.83, 0^\circ \leq \alpha \leq 180^\circ )</td>
</tr>
<tr>
<td>Lam et al. (2008)</td>
<td>Numerical</td>
<td>100&amp;200</td>
<td>( 1.6 \leq P \leq 5, \alpha = 0^\circ )</td>
</tr>
<tr>
<td>Lam and Zou (2010)</td>
<td>Numerical</td>
<td>200</td>
<td>( 1.6 \leq P \leq 5, \alpha = 0^\circ )</td>
</tr>
</tbody>
</table>

Table 1-2 A summary of selected studies on steady flow around four-circular cylinder

Compared with that of a single cylinder, research concerning steady flow around multiple cylinders is relatively limited, especially for more than two structures. Selected researches on steady flow around four cylinders are listed in Table 1-2.

Sayers (1988, 1990) carried out a series of experimental studies on flow past four equally spaced cylinders in square arrangements with spacing ratio ranging from 1.1 to 5 at \( Re = 3 \times 10^4 \) and concluded that vortex shedding frequency was largely influenced by the spacing ratio and the incoming flow direction. When the spacing ratio was larger than 4, the Strouhal number of each cylinder was equal to that of a single cylinder, indicating the weak interference among the cylinders at large spacing ratios. The flow pattern and the force coefficients were also strongly influenced by the orientation of the cylinder array. At small spacing ratios (1.1~3), a small change in the direction of the free stream velocity may lead to significant changes in vortex shedding frequency and force coefficients.

Farrant et al. (2000) numerically investigated the laminar 2-D flow past four equally
spaced cylinders at Re = 200 in two orientations, i.e., the in-line alignment and the alignment angle of 45° using a cell boundary element method. Two spacing ratios, namely, 3 and 5 were considered and it was found that, in the in-line arrangement, the in-phase vortex-shedding mode occurred when the spacing ratio was 3, while anti-phase vortex-shedding mode dominated at the spacing ratio of 5.

By conducting flow visualization studies, Lam et al. (2003a) and Lam and Lo (1992) found suppression of vortex shedding from the upstream cylinders at spacing ratios less than 3.94, which was defined as the critical spacing ratio for flow pattern transformation. Lam and Fang (1995), Lam et al. (2003b) and Lam and Zou (2007) further studied the effects of spacing ratio on the pressure distributions and the lift and drag coefficients at sub-critical Reynolds numbers. Numerical studies by Lam et al. (2008) were focused on the prediction of the vortex shedding flow pattern based on 2-D studies. They found that 2-D simulations cannot adequately represent flow around four cylinders and there was a large discrepancy between experimental measurements and 2-D numerical results of the critical spacing ratios, which have been remedied and well captured by the 3-D simulations later by Lam and Zou (2010).

1.1.3 Oscillatory flow around a single cylinder

Sinusoidally oscillatory flow around a circular cylinder has also been studied extensively for decades. Measurements on loads induced by oscillatory flow on a single cylinder (Maull and Milliner, 1978, Williamson, 1985, Bearman et al., 1985, Obasaju et al., 1988, Sarpkaya, 2002, Saghaian et al., 2003) have direct relevance to engineering applications. The flow at relatively low frequencies and amplitudes of flow oscillation (Honji, 1981, Tatsuno and Bearman, 1990, Dütsch et al., 1998, Elston et al., 2006) presents a tapestry of flow patterns and thus attract much academic interest. The oscillatory flow around a circular cylinder is mainly governed by two dimensionless parameters, namely the Keulegan-Carpenter number $KC$ and the Reynolds number $Re$. The $KC$ is conventionally defined as $KC = U_mT / D$, where $U_m$ is the amplitude of velocity oscillation and $T$ is the period of the oscillating flow. The ratio of Re and $KC$, known as the frequency parameter or Stokes number ($\beta$) is also often referred in literature.

Comprehensive flow features induced by sinusoidal oscillations of a circular cylinder in otherwise stationary fluid at low $KC$ and low $\beta$ were experimentally identified by
Tatsuno and Bearman (1990). Eight flow regimes were classified within $1.6 \leq KC \leq 15$ and $5 \leq \beta \leq 160$. Among the flow regimes, flow in regime A and $A^*$ is two-dimensional and symmetric to the direction of motion, with vortex shedding occurring in regime A but not in $A^*$ (Anagnostopoulos and Minear, 2004). Regime B is featured with the so-called ‘streaked flow’ along the axis of the cylinder, which is composed of equally spaced streaks of chains of separated flow structures, each in a form similar to a mushroom (Honji, 1981, Sarpkaya, 1986, An et al., 2011). Flows in regimes C ~ G are apparently three-dimensional. A key observation in regime C is that the vortex fields are not synchronized with the oscillating period, and are rather rearranged into large vortices with a secondary period before emanating in the directions of motion (Elston et al., 2006). The transverse vortex streets are found in regimes D and E, where flow are obliquely convected to one side of the axis of oscillation (Lin et al., 1996). Irregular switching of the convection direction is evolved in regime E (Dütsch et al., 1998, Uzunoğlu et al., 2001). Diagonal double-pair vortices are featured in regime F (Scandura et al., 2009).

Many experimental tests and numerical studies have been carried out within the parameter ranges covered in the map of regimes drawn by Tatsuno and Bearman (1990) and significant amount of knowledge has been gained through these studies, which has been reviewed by Bearman (1984) and Elston et al. (2006).

Nehari et al. (2004) compared the 2-D and 3-D numerical results on regimes D and F. It was reported that in-line force component is only weakly affected by the three-dimensional effect. It was found the cross-sectional vortex streets appear to be unstable and switching from a pattern to its mirror-image mode but are related to a two-dimensional instability and can be reproduced by pure two-dimensional simulations. The predictions flow features in the 3-D regimes by 2-D numerical models were also reported by several other studies (Lin et al., 1996, Uzunoğlu et al., 2001, Anagnostopoulos and Dikarou, 2011, Zhao and Cheng, 2014).

1.1.4 Oscillatory flow around multiple cylinders

In contrast, studies on oscillatory flow around multiple cylinders (two or more) have not been documented extensively in literature. As a matter of fact, cylindrical structure elements are commonly arranged in groups due to functional requirements in engineering applications. For example use of four circular cylinders in a square
arrangement can be found in offshore Tension Leg Platforms (TLP). Moreover rich physics has also been discovered in steady flow around multiple cylinders at close proximity. As discussed above, when two or more circular cylinders are arranged close to each other, the proximity interference and the wake interference lead to more complex flow features than those in the wake of a single cylinder (Zdravkovich, 1987, Hu and Zhou, 2008a). It is anticipated that oscillatory flow around multiple structures likely shares these interferences with steady flow. However our understanding is limited by a scarcity of research on the subject. This motivates the present study of oscillatory flow around four cylinders of a square arrangement.

Williamson (1985) carried out an experimental study to investigate the synchronization of vortex shedding of two oscillatory cylinders in still fluid and measured the forces induced by oscillating flow on the two cylinders. Uzunoğlu et al. (2001) investigated the flow fields and force coefficients for two cylinders in side-by-side and tandem configurations. It is only until recently that oscillatory flow around two or four cylinders has attracted some research interests, mainly based on 2-D numerical models. Chern et al. (2010) and Chern et al. (2013) simulated oscillatory flow past two side-by-side square cylinders and four circular cylinders in staggered and in-line arrangements, respectively. It was found the gap flow between the cylinders plays a significant role on the flow fields and hydrodynamic forces. Yang et al. (2013) investigated oscillatory flow around a pair of cylinders of unequal diameters based a 2-D model. The influence of the gap ratio and the positional angle on the flow field and hydrodynamics forces was investigated. There has not been a systematic study on flow regimes around four cylinders, to the best knowledge of the authors. Zhao and Cheng (2014) investigated oscillatory flow around a two-cylinder system in both side-by-side and tandem arrangements at two Reynolds numbers of 150 and 100, by solving the two-dimensional Navier-Stokes equations with a finite element method. The authors identified several combined flow regimes comprised of the flow regimes observed around a single cylinder, as well as new flow features, such as Gap Vortex Shedding (GVS), where the vortices only shed from the gap side of the system.

1.2 Research goals

It is seen from the above literature review that there are considerable gaps between studies of steady & oscillatory flow around a single cylinder and multiple cylinders. It is the aim of the present research to present the flow features around multiple cylinders
and force coefficients induced on these structures in close proximity. The main research goals are,

1. To investigate the influence of pitch distances and alignment angles on the flow regimes and force distribution on two staggered cylinders;
2. To classify the flow features under the influence of Reynolds number and gap distances around four circular cylinders in close proximity;
3. To study the influences of Reynolds number on the flow features and force behaviours of four circular cylinders;
4. To model and classify the sinusoidally oscillatory flow fields around four circular cylinders in the in-line square arrangement.

1.3 Outline

This thesis comprises six chapters and one appendix. The reminder of the thesis has been organized in the following manner.

Steady uniform flow around two identical circular cylinders at a subcritical Reynolds number is studied in Chapter 2. Various pitch distances between the cylinders and cylinders arrangements to the cross flow are investigated.

Simulations on steady flow around a four-cylinder array in a square arrangement are conducted in Chapter 3, which is highlighted by a total of seven flow regimes with distinguished flow features. These flow patterns are identified and mapped on the plane of Reynolds number and pitch distance.

Three-dimensional (3-D) simulations are carried out in Chapter 4 to study the vortex shedding flow in the wake of four circular cylinders in a square configuration at one pitch distance where four distinct flow regimes are identified, along with significant changes in the force coefficients, wake formation length and phase angle of the lift coefficients on the downstream cylinders when the flow transits from one regime to another. Physical mechanisms responsible for different flow regimes are proposed and discussed in details.

Sinusoidally oscillatory flow around four circular cylinders in the in-line square arrangement is modeled in Chapter 5 at oscillations with relatively low frequency and
low amplitude. The primary aim is to investigate the influence of cylinder proximity on flow regimes. A captivating set of flow patterns is observed and identified, including six types of reflection symmetry to the axis of oscillation, two types of spatio-temporal symmetry and a series of symmetric breaking flow patterns.

Chapter 6 summarizes the main outcomes of this research, along with suggestions for future studies.

Finally, appendix A provides a guide on optimizing CPU cores when CFD simulations are carried out on parallel clusters.

1.4 References


Chapter 2

Three-dimensional numerical simulations of steady flow past two cylinders in staggered arrangements

Numerical simulations of steady flow past two cylinders in staggered arrangements

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²School of Computing, Engineering and Mathematics, University of Western Sydney, Locked Bag 1797, Penrith, NSW 2751, Australia

This paper presents a numerical study on steady flow around two identical circular cylinders of various arrangements at a low subcritical Reynolds number (Re=10³). The ratio of center-to-center pitch distance (P) to the diameter of the cylinder (D) ranges from 1.5 to 4, and the alignment angle (α) between the two cylinders and the direction of the cross flow varies from 0° to 90°. The detailed flow information obtained from direct numerical simulation allows a comprehensive interpretation of underlying physics responsible for some interesting flow features observed around two staggered cylinders. Four distinct vortex shedding regimes are identified and it is demonstrated that accurate classification of vortex shedding regimes around two staggered cylinders should

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consider the combination of the flow visualization and the analyses of lift forces and velocity signal in the wake. It is revealed that the change in pressure distribution, as a result of different vortex shedding mechanism, leads to a variety of characteristics of hydrodynamic forces on both cylinders, including negative drag force, attractive and repulsive lift forces. Two distinct vortex shedding frequencies are identified and are attributed to the space differences based on the flow structures observed in the wake of the cylinders. It is also found that three-dimensionality of the flow in the gap and the shared wake regions is significantly weakened by cylinder arrangement in two of the classified flow regimes; however, active wake interaction at large $\alpha$ does not clearly increase the three-dimensionality of the flow.

2.1 Introduction

Flow around a cylinder undergoes substantial changes when it is a member of a group of cylinders in close proximity. The simplest case of a group of cylinders is two identical circular cylinders with various arrangements, which have been widely studied in the recent decades. Figure 2-1 shows a sketch of two identical cylinders in a staggered arrangement, which is the object of this study. Flow characteristics around the two-cylinder system are understandably dependent on the pitch ratio $P/D$, flow alignment angle $\alpha$ and Reynolds number. The Reynolds number is defined as $\text{Re} = U_\infty D/\nu$, where $U_\infty$ is the free stream velocity and $\nu$ is the kinematic viscosity of the fluid.

A variety of flow interferences have been studied around two cylinders, such as interactions between the shear layers, vortex shedding and the Kármán vortex streets at the downstream. These interferences present a rich spectrum of flow features, accompanied by a great diversity of forces and other parameters on the structures. Zdravkovich (1977; 1987) classified the flow field into two basic types of interference based on the location of the downstream cylinder with respect to the upstream one, i.e., the wake interference and the proximity interference. For the convenience of discussions, the upstream and downstream cylinders are referred to as Cyl_up and Cyl_down respectively, here afterwards. The wake interference is obvious when the two cylinders are in a tandem arrangement ($\alpha = 0^\circ$), where one cylinder is submerged in the wake of the other. The proximity interference occurs when the two cylinders are close to one another but neither is in the wake of the other, as in the side-by-side configuration ($\alpha = 90^\circ$). Both wake and proximity interferences may present in flow around two cylinders in a staggered arrangement.
Figure 2-1 A schematic representation of flow around two cylinders. The approaching uniform flow $U_\infty$ is from left to right. The cylinders of diameter $D$ are represented by two circles. The flow approach angle is defined relative to the line linking the centres of the cylinders.

Extended body regime $\ 1 < \frac{P}{D} < 2$
Reattachment regime $\ 1.2 < \frac{P}{D} < 5.0$
Co-vortex shedding regime $\ \frac{P}{D} > 3.4$

Figure 2-2 A broad classification of the flow regimes for two tandem circular cylinders in steady current, reproduced from Zdravkovich (1987) and Xu and Zhou (2004). The approach flow is from left to right.

Igarashi (1981; 1984) identified eight different flow regimes for flow past two tandem cylinders, which were later narrowed down into three (Xu and Zhou 2004; Zdravkovich 1987; Sumner 2010), namely, (i) extended-body regime at small pitch ratios, where the two cylinders behave as a single bluff-body; (ii) reattachment regime at intermediate pitch ratios, where the separated shear layers from the upstream cylinder reattach to the downstream cylinder; and (iii) co-shedding regime at large pitch ratios, where Kármán vortices are shed from both cylinders. Schematic profiles of these flow regimes based on Zdravkovich (1987) and Xu and Zhou (2004) are shown in Figure 2-2. The overlaps of pitch ratios shown in Figure 2-2 for different flow regimes are likely due to the differences in Reynolds number in the studies by Zdravkovich (1987) and Xu and Zhou (2004).

Flow around two side-by-side cylinders has also been studied extensively. Three flow regimes around two side-by-side cylinders have been identified by Sumner (2010). They are (i) single-bluff-body at small pitch ratios, where the two cylinders are close enough to act as a single structure; (ii) biased flow regime at intermediate pitch ratios, where the gap flow deflects towards one of the two cylinders, resulting in an asymmetric flow field; and (iii) parallel vortex streets at large pitch ratios, where a symmetric Kármán vortex street is observed behind each cylinder. The aforementioned
flow regimes for two side-by-side cylinders are illustrated in Figure 2-3. Again, the flow features at two adjacent flow regimes are sensitive to Re.

Figure 2-3 A broad classification of flow regimes for two side-by-side circular cylinders in steady current, reproduced from Sumner (2010). Approach flow is from left to right.

<table>
<thead>
<tr>
<th>Research</th>
<th>Re</th>
<th>Arrangement</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Staggered</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mittal et al. (1997)</td>
<td>$10^2, 10^3$</td>
<td>$T/D = 0.7$ and $P/D = 5.5$</td>
<td>2-D, FEM</td>
</tr>
<tr>
<td>Jester and Kallinderis (2003)</td>
<td>$80 ~ 10^4$</td>
<td>$0 \leq L/D \leq 7, 0 \leq T/D \leq 4$</td>
<td>2-D, FEM</td>
</tr>
<tr>
<td>Akbari and Price (2005)</td>
<td>800</td>
<td>$1 &lt; P/D \leq 4, \alpha = 0^\circ ~ 90^\circ$</td>
<td>2-D, NVM</td>
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<tr>
<td>Carmo et al. (2008)</td>
<td>200 ~ 350</td>
<td>$L/D = 5, 0 \leq T/D \leq 3$</td>
<td>3-D, Spectral/hp</td>
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<tr>
<td>Lee and Yang (2009)</td>
<td>$10^3$</td>
<td>$0 \leq L/D \leq 5, 0 \leq T/D \leq 5$</td>
<td>2-D, IBM</td>
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<tr>
<td>Present study</td>
<td>$10^3$</td>
<td>$1.5 \leq P/D \leq 4, \alpha = 0^\circ ~ 90^\circ$</td>
<td>3-D&amp;2-D, FVM</td>
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<td><strong>Tandem</strong></td>
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<td></td>
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</tr>
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<td>Farrant et al. (2000)</td>
<td>200</td>
<td>$L/D = 5.0, T/D = 0$</td>
<td>2-D, Cell BEM</td>
</tr>
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<td>Papaioannou et al. (2006)</td>
<td>$10^3 \sim 10^4$</td>
<td>$1.1 \leq L/D \leq 5.0, T/D = 0$</td>
<td>3-D, Spectral/hp</td>
</tr>
<tr>
<td>Carmo and Meneghini (2006)</td>
<td>160 ~ 320</td>
<td>$1.5 \leq L/D \leq 8.0, T/D = 0$</td>
<td>3-D, Spectral/hp</td>
</tr>
<tr>
<td>Kitagawa and Ohta (2008)</td>
<td>$2.2 \times 10^4$</td>
<td>$2.0 \leq L/D \leq 5.0, T/D = 0$</td>
<td>3-D, FDM, LES</td>
</tr>
<tr>
<td>Carmo et al. (2010a)</td>
<td>50 ~ 500</td>
<td>$2.3 \leq L/D \leq 5.0, T/D = 0$</td>
<td>3-D, Spectral/hp</td>
</tr>
<tr>
<td>Carmo et al. (2010b)</td>
<td>200 ~ 350</td>
<td>$1.2 \leq L/D \leq 10.0, T/D = 0$</td>
<td>3-D, Spectral/hp</td>
</tr>
<tr>
<td><strong>Side-by-side</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Meneghini et al. (2001)</td>
<td>100, 200</td>
<td>$L/D = 0, 1.5 \leq T/D \leq 4.0$</td>
<td>2-D, FEM</td>
</tr>
<tr>
<td>Chen et al. (2003)</td>
<td>750</td>
<td>$L/D = 0, T/D = 1.7, 3.0$</td>
<td>3-D, FVM, LES</td>
</tr>
<tr>
<td>Afgan et al. (2011)</td>
<td>3900</td>
<td>$L/D = 0, T/D = 1.0 ~ 5.0$</td>
<td>3-D, FVM, LES</td>
</tr>
<tr>
<td>Shao and Zhang (2008)</td>
<td>5800</td>
<td>$L/D = 0, 1.0 \leq T/D \leq 6.0$</td>
<td>3-D, FVM, LES</td>
</tr>
<tr>
<td>Mizushima and Ino (2008)</td>
<td>$\leq 80$</td>
<td>$L/D = 0, 1.3 \leq T/D \leq 2.0$</td>
<td>2D</td>
</tr>
</tbody>
</table>

Table 2-1 Selected numerical studies of two staggered circular cylinders in cross-flow (cylinders in the tandem and side-by-side configurations are also included). 2-D = two dimensional study; 3-D = three dimensional study; FEM = Finite Element Method; NVM = Numerical Vortex Method; IBM = Immersed Boundary Method; FDM = Finite Difference Method; LES = Large Eddy Simulation.

Flow around two cylinders in staggered arrangement is also commonly encountered in practical applications. The side-by-side and tandem arrangements of two cylinders mentioned above are two special cases of the two staggered cylinders (Hu and Zhou 2008a). Most experimental studies about flow past two staggered cylinders were carried out at the high end of the subcritical flow regime for an isolated cylinder. Classifications of flow regimes around two staggered cylinders have been discussed in several studies. By examining pressure distributions and flow fields, Gu and Sun (1999) identified three
flow interference patterns for \( P/D \) ranging from 1.1 to 3.5 and \( \text{Re} \) ranging from \( 2.2 \times 10^5 \) to \( 3.3 \times 10^5 \), which are referred to as wake interference, shear layer interference and neighborhood interference. Sumner et al. (2000) identified nine different flow patterns through detailed flow visualizations, which include shear layer reattachment and separation, vortex pairing, synchronization and impingement. The studies by Hu and Zhou (2008a; 2008b) were focused on the evolution of flow structures, Strouhal number in the wake, and the heat and momentum transfer in the flow in the wake of two staggered cylinders. The wake flow fields at \( \text{Re} = 7000 \) were classified into four regimes based on flow visualizations and Strouhal numbers, namely the two single-wake regimes and two twin-wake regimes. It is clear from the existing studies that the flow behavior is heavily dependent on the arrangement of the cylinders. Sumner (2010) provided a comprehensive review on the experimental studies on flow around two staggered cylinders. It was suggested that the flow patterns around two staggered cylinders can be classified into three broad regimes, which are named as single-bluff-body flow regimes, small-incidence-angle flow regimes, and large-incidence-angle flow regimes (Sumner 2010). The flow regimes shown in Figure 2-2 and Figure 2-3 are the fundamental elements of the flow patterns identified for two staggered cylinders so far.

Numerical studies of flow around two staggered cylinders have been rather limited. Most available numerical studies are carried out at low \( \text{Re} \) values using two-dimensional (2-D) models. This was likely attributed to the limitation of available computational power to resolve the complicated flow field. The only 3-D numerical study on two staggered cylinders (Carmo et al. 2008) focuses on Floquet stability analysis of wake transitions from 2-D to 3-D. No 3-D numerical work has been carried out to investigate the effect of pitch ratio and alignment angle on vortex shedding and hydrodynamic forces on two staggered cylinders, although this has been carried out in many 2-D numerical simulations (Akbari and Price 2005; Jester and Kallinderis 2003; Lee and Yang 2009; Mittal et al. 1997). Table 2-1 summarizes the numerical investigations on steady flow around two staggered cylinders, known to the authors, together with selected numerical studies on two cylinders in both side-by-side and tandem arrangements.
With the rapid increase in computational power in recent years, numerical studies have been widely used to understand underlying physics of many complex flow problems due to their unique attributes over experimental investigations. To this end, this work presents a numerical study on the effect of pitch ratio and alignment angle on steady flow around two staggered cylinders near the lower end of the subcritical Re regime with Re = 10^3. Figure 2-4 presents a summary of the parameter space covered in this study. A total of four pitch ratios (1.5, 2, 3 and 4) and nine alignment angles are considered in the present study. The selected pitch ratios cover all possible flow regimes (based on the knowledge derived from existing experimental studies), while the alignment is chosen based on the consideration that the forces and flow features are sensitive to \( \alpha \) at small alignment angles. Although much of the result is based on 3-D numerical simulations, 2-D simulations are also provided to explain some fluid features. The 2-D simulation is not expected to capture the physics of the flow at Re = 10^3, but it numerically disposes the flow in the third direction thus provides a way to estimate the influence of 3-D flow. The remainder of the paper is organized in the following manner. In § 2, the numerical model and model validations are presented, while § 3 presents the forces and pressure distributions on both cylinders. Vortex shedding frequencies are discussed in § 4, followed by the discussion on flow characteristics in § 5. Finally, the conclusions are given in § 6.
2.2 Numerical model

2.2.1 Numerical method

Steady uniform flow around the two-cylinder system is simulated by solving the incompressible Navier–Stokes (NS) equations. The vector form of 3-D NS equations in the Cartesian coordinate system are expressed as

\[
\begin{align*}
\frac{\partial \vec{U}}{\partial t} + \nabla \cdot (\vec{U} \vec{U}) - \nabla \cdot p\nabla \vec{U} &= -\frac{\nabla p}{\rho} \\
\nabla \cdot \vec{U} &= 0
\end{align*}
\]

(2-1)

(2-2)

where the velocity vector \(\vec{U}\) has three components \(U_x, U_y,\) and \(U_z\) in the \(x, y,\) and \(z\)-directions respectively, \(t\) the time, \(\rho\) is the density of the fluid and \(p\) is the pressure. The NS equations are solved by the Open source Field Operation and Manipulation (OpenFOAM®) C++ libraries, which is a free source CFD package developed by OpenCFD Ltd. The finite volume method (FVM) is used in the solver and the pressure-velocity coupling is based on the Pressure Implicit with Splitting of Operators (PISO) method. The convection terms are discretised using the Gauss cubic scheme, while the Laplacian and pressure terms in the momentum equations are discretised using the Gauss linear scheme. The Euler implicit scheme is adopted for the temporal discretisation.

The initial values for flow velocity and pressure in the whole domain are set to zero and boundary conditions for the governing equations are: (i) at the inlet, a uniform velocity \(U_\infty\) with certain incident angle is given and the pressure gradient in the streamwise direction is specified as zero; (ii) at the outlet, the pressure is set to zero and the velocity gradients in the streamwise direction are zero; (iii) no-slip boundary condition is adopted on the cylinder surfaces and (iv) symmetry boundary conditions are applied at two lateral boundaries that are perpendicular to the cylinders.

2.2.2 Mesh dependency and model validation

A cubic computational domain as shown in Figure 2-5(a) is used in this study. In the present numerical simulation, the alignment angle is varied by changing the flow direction. Two inlet boundaries are set for simulating flow past two cylinders at oblique approaching angles. The two inlet boundaries are 16\(D\) away from Cyl_up and the two outlet boundaries are 29\(D\) away from Cyl_down. This leads to a blockage ratio ranging
from 2.2% at $\alpha=0^\circ$ to 4.4% at $\alpha=90^\circ$. The height of the domain ($H$) in the spanwise direction of the cylinders is $10D$. The size of the computational domain is selected based on a domain size dependency study and the experience from Barkley and Henderson (1996). The choice of the spanwise length $L_z$ of the domain was based on the considerations of both efficiency and accuracy and the length of $3\pi$ was found to be sufficient for capturing the flow variation in the spanwise direction of the cylinders (Henderson and Karniadakis 1995; Lei et al. 2001; Labbé and Wilson 2007).

![Figure 2-5 A schematic representation of the computational domain and mesh distributions of the two-circular cylinder system.](image)

Figure 2-5 A schematic representation of the computational domain and mesh distributions of the two-circular cylinder system. (a), computational domain, and the inlet flow $U_\infty$ is from bottom left to top right; (b) 3-D view of the mesh around two cylinders; (c), close view of the mesh surrounding the cylinders, which consist layers of structured quadrilateral cells, followed by unstructured cells. (d), local view of the mesh surrounding the cylinders, where the unstructured cells shown in (c) is followed by structured quadrilateral mesh again.

The computational mesh is generated using an open source generator named Gmsh (Geuzaine and Remacle 2009). Unstructured hexahedron finite volumes are used in the numerical simulations. Each computational mesh consists of structured cells near the cylinder surfaces (Figure 2-5, c) and also in the far field (Figure 2-5, b), and unstructured cells in a rectangular area as shown in Figure 2-5 (d). The size of the
structured cells around the cylinder surfaces is chosen based on a mesh dependence study to ensure the numerical accuracy at Re=10^3. The rectangular domain that bounds the unstructured cells is chosen based on the criteria that the distance between its outer boundaries to the nearest cylinder is 4D. Small unstructured cells are deliberately distributed in areas where velocity and pressure gradients are anticipated to be high. The 3-D mesh is formed by extending the two dimensional mesh as shown in Figure 2-5 (c&d) in equidistance in the z-direction of 100 layers.

To investigate the effect of mesh density on the accuracy of the solution, a mesh dependence study is carried out for flow past a single cylinder with four different meshes at Re=10^3 in Table 2-2. The four meshes differ from each other in the mesh density near the cylinder surface as well as in the wake region. The radial sizes of the cell on the cylinder surface for mesh 1, 2, 3 and 4 are 1×10^{-2}D, 5×10^{-3}D, 1×10^{-3}D and 5×10^{-4}D, respectively. The cells number of the mesh ranges from 1.4 million to 3.4 million. The corresponding non-dimensional distances from the first nodal point to the cylinder surface, \( y^+ = u_f \Delta / \nu \) are also included in Table 2-2, where \( \Delta \) is the distance from the cylinder surface and \( u_f \) is the friction velocity. The computational domain size around a single cylinder is 32D×45D, and the mesh density and distribution are similar to those shown in Figure 2-5.

To ensure the establishment of a fully developed flow field, each computation is run for at least \( t^* = 400 \), where \( t^* \) is non-dimensional time defined as \( t^* = U_\infty t / D \). Numerical results of the force coefficient and Strouhal number for a single cylinder are listed in Table 2-2 along with published experimental and three-dimensional numerical results. Here, \( C_D \) is the drag coefficient, \( \bar{C}_D \) is the mean drag coefficient; \( C_L \) is the lift coefficient, \( C'_L \) is root-mean-square (rms) lift coefficient; and \( St \) is Strouhal number, which are defined as,

\[
C_D = \frac{F_x}{\rho U_\infty^2 D/2}, \quad C_L = \frac{F_y}{\rho U_\infty^2 D/2}, \quad St = \frac{f_s D}{U_\infty}
\]  

(2-3)

where \( F_x \) is the drag force in the direction of approaching flow and \( F_y \) is the lift force perpendicular to the approaching flow, \( f_s \) is the frequency of the fluctuating lift force, obtained by performing the Fast Fourier Transition (FFT) analysis of the lift coefficient. \( C_p \) is pressure coefficient and is defined as,
\[ C_p = \frac{P - P_{\infty}}{\rho U_{\infty}^2 / 2} \]  

(2-4)

where \( P_{\infty} \) is the free stream pressure. \( C_{pb} \) is the pressure coefficient at the base point.

The mesh dependence study shows that the numerical results are less sensitive to the mesh density if a mesh as fine as (or finer than) mesh 3 is chosen. The overall numerical results from all meshes appear to agree reasonably well with the published results.

<table>
<thead>
<tr>
<th>Sources</th>
<th>Mesh</th>
<th>( \Delta/D )</th>
<th>( y^+ )</th>
<th>( L_z/D )</th>
<th>( \Delta z/D )</th>
<th>( -C_{pb} )</th>
<th>( \bar{C}_D )</th>
<th>( C_t )</th>
<th>( St )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>mesh 1</td>
<td>( 1 \times 10^{-2} )</td>
<td>1.05</td>
<td>10</td>
<td>0.1</td>
<td>0.97</td>
<td>1.15</td>
<td>0.26</td>
<td>0.208</td>
</tr>
<tr>
<td></td>
<td>mesh 2</td>
<td>( 5 \times 10^{-3} )</td>
<td>0.51</td>
<td>10</td>
<td>0.1</td>
<td>0.95</td>
<td>1.13</td>
<td>0.24</td>
<td>0.213</td>
</tr>
<tr>
<td></td>
<td>mesh 3</td>
<td>( 1 \times 10^{-3} )</td>
<td>0.105</td>
<td>10</td>
<td>0.1</td>
<td>0.90</td>
<td>1.09</td>
<td>0.20</td>
<td>0.215</td>
</tr>
<tr>
<td></td>
<td>mesh 4</td>
<td>( 5 \times 10^{-4} )</td>
<td>0.051</td>
<td>10</td>
<td>0.1</td>
<td>0.89</td>
<td>1.08</td>
<td>0.20</td>
<td>0.215</td>
</tr>
<tr>
<td><strong>Numerical</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naito and Fukagata (2012)</td>
<td>mesh 1</td>
<td>( 1 \times 10^{-2} )</td>
<td>1.05</td>
<td>2( \pi )</td>
<td>0.1</td>
<td>0.92</td>
<td>1.09</td>
<td>0.21</td>
<td>0.21</td>
</tr>
<tr>
<td>Henderson and Karniadakis (1995)</td>
<td>-</td>
<td>-</td>
<td>4( \pi )</td>
<td>0.39</td>
<td>1.12</td>
<td>1.20</td>
<td>0.21</td>
<td>0.211</td>
<td></td>
</tr>
<tr>
<td>Papaioannou et al. (2006)</td>
<td>-</td>
<td>-</td>
<td>3( \pi )</td>
<td>0.15</td>
<td>0.82</td>
<td>1.03</td>
<td>-</td>
<td>- 0.216</td>
<td></td>
</tr>
<tr>
<td>Zhao et al. (2009)</td>
<td>mesh 1</td>
<td>( 1 \times 10^{-3} )</td>
<td>0.105</td>
<td>9.6</td>
<td>0.1</td>
<td>1.08</td>
<td>1.17</td>
<td>0.34</td>
<td>0.210</td>
</tr>
<tr>
<td><strong>Experimental</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Williamson and Roshko (1990)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.81</td>
<td>-</td>
<td>-</td>
<td>- 0.21</td>
<td></td>
</tr>
<tr>
<td>Norberg (2002)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.81</td>
<td>0.91 *</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table 2-2 Mesh details and the influence of mesh size on simulation results for a single cylinder at \( Re = 10^7 \)

\* mean pressure drag coefficient

Figure 2-6 Comparison on the pressure coefficient on the surface of the cylinder from Mesh 3 with data of Norberg (2002) and Naito and Fukagata (2012).

In addition to the comparison of the integrated flow quantities, distribution of pressure coefficient \( (C_p) \) around the cylinder based on mesh 3 are compared with available data from literature in Figure 2-6. Good agreement of the present \( C_p \) values with experimental results by Norberg (2002) and numerical results from Naito and Fukagata (2012) is observed. All three studies suggest the minimum pressure coefficient occurs at about 75° measuring from the stagnation point.
The mesh resolution in the cylinder wake region is examined by comparing the velocity profile along $x/D = 5$ with spectral element numerical result from Henderson and Karniadakis (1995) in Figure 2-7. Both streamwise and normal velocity profiles are consistent with the published data, and active velocity fluctuations are observed within a 2$D$ distance in each side of the cylinder. The increased mesh resolution in the wake in mesh 2, 3 and 4 leads to less than 2% difference in the mean velocity profile; therefore further increasing in grid number makes little change on the flow field, while only requires more the computational effort.

![Figure 2-7 Variation of mean and fluctuation velocity profile for $U_x$ and $U_y$ at $x/D = 5$; ---/○, mean velocity; ---/●, velocity fluctuation; lines, present study; symbols, Henderson and Karniadakis (1995).](image)

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Domain size</th>
<th>$C_{D1}$</th>
<th>$C_{D2}$</th>
<th>$C_{L1}$</th>
<th>$C_{L2}$</th>
<th>$S_{t1}$</th>
<th>$S_{t2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>16$D$/29$D$ (32$D$$\times$45$D$)</td>
<td>1.05</td>
<td>1.22</td>
<td>-0.26</td>
<td>0.36</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>6</td>
<td>32$D$/58$D$ (64$D$$\times$90$D$)</td>
<td>1.02</td>
<td>1.19</td>
<td>-0.25</td>
<td>0.35</td>
<td>0.32</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 2-3 Influences of the blockage effect on the simulation of two side-by-side cylinders at $Re=10^3$; subscript 1&2 represent the top and bottom cylinders respectively.

The blockage effect of the cylinders on the numerical results is investigated by simulating flow past two side-by-side cylinders ($\alpha = 90^\circ$) at a pitch ratio of 1.5, where the blockage ratio (4.4%) is the maximum among all the cases investigated in this study. At such a small spacing, it is anticipated that the blockage effect is more pronounced because the two cylinders usually behave as a single structure of a characteristic width of 3.5$D$. Simulations are conducted using two domain sizes with similar mesh densities to that used in mesh 3. The first domain size is the same as that used in Table 2-2 and is named mesh 5. The second domain size doubles that of mesh 5 and is named mesh 6. The simulation results are compared in Table 2-3, where the differences between the two cases are negligible, with the maximum difference of about 4% in the mean lift. The flow patterns (not shown here) based on the two cases are also found to be similar.
For a cross comparison, the domain size used in the present study is larger than those used by Lam et al. (2010) for the study on flow around four cylinders. In light of these evidences, the computational domain sizes are considered to be adequate for the purpose of this study.

To further validate the present numerical model, the calculated drag coefficients for two tandem circular cylinders at Re=10^3 are compared with numerical results data by Papaioannou et al. (2006) in Figure 2-8. The mean $C_D$ for an isolated cylinder is also given in Figure 2-8 as a reference. The agreement between the two sets of numerical results is excellent. For all pitch ratios, the mean $C_D$ of each of the two cylinders typically remains smaller than that of an isolated cylinder. When the pitch ratio is small (<3.5) vortex shedding occurs only in the wake of Cyl_down. $C_D$ of Cyl_up decreases slowly with the pitch ratio until a value between 3.5 and 4.0, where the co-vortex shedding regime starts (i.e., vortex shedding occurs from both cylinders), resulting in a sudden jump in $C_D$ of Cyl_up to the value close to that of a single cylinder. The mean drag on Cyl_down remains negative for $P/D \leq 3.5$ (within the extended-body and reattachment regimes). It experiences a more prominent jump to be positive after the occurrence of vortex shedding from Cyl_up. This suggests that the sudden increase of the drag coefficient on the downstream cylinder is a good indicator of the occurrence of vortex shedding from upstream cylinder.

The 2-D numerical model is also validated by comparing the present results with the published data for flow around two tandem cylinders at Re=10^3 and $P/D = 3$ in Table
The Strouhal numbers for the two cylinders are found to be the same. It is seen that the present results agree well with those reported in literature.

<table>
<thead>
<tr>
<th>Source</th>
<th>$C_{D1}$</th>
<th>$C_{D2}$</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present study</td>
<td>1.43</td>
<td>0.13</td>
<td>0.225</td>
</tr>
<tr>
<td>Jester and Kallinderis (2003)</td>
<td>1.42</td>
<td>0.16</td>
<td>NA</td>
</tr>
<tr>
<td>Papaioannou et al. (2006)</td>
<td>1.44</td>
<td>0.13</td>
<td>0.222</td>
</tr>
</tbody>
</table>

Table 2-4 Comparison of the present 2-D results with published data for flow around two tandem cylinders at $Re=10^3$; subscript 1&2 in the force coefficients represent the up- and downstream cylinders, respectively.

These validation results suggest that a domain size with $16D$ at inlet and $29D$ at outlet is sufficient for modelling steady flow around two cylinders at $Re=10^3$; and the numerical results are mesh independent if the mesh is equivalent to or denser than those in mesh 3. Thus, the computational domain for two staggered cylinders is meshed on the basis of mesh 3 in Table 2-2, resulting in a total number of cells varying from 4,086,300 at $P/D = 1.5$ to 4,833,500 at $P/D = 4$.

### 2.3 Pressure and force

#### 2.3.1 Pressure distribution

Firstly, the effect of staggered arrangements on pressure distribution around two cylinders is presented. The variations of the pressure coefficient $C_p$ with the position angle $\theta$ for flows with two pitch ratios of $P/D = 1.5$ & 3 and four selected alignment angles from 3-D simulations are shown in both Cartesian and polar coordinates in Figure 2-9, where illustrated are the sectional averaged $C_p$ at the middle cross section plane ($z = 5D$). The position angle $\theta$ along a cylinder surface is defined in Figure 2-9 (a). The pressure in the polar coordinate system is negative inside the cylinder surface and positive outside. It should be clarified that in the following discussion, the base point on a cylinder surface is defined as the extreme aft point ($\theta = 0^\circ$) in the flow direction, regardless of the influence from the other cylinder, and the pressure at the base point is defined as base pressure, $C_{pb}$; while the stagnation point is referred to as the point with the maximum pressure coefficient, which is referred as stagnation pressure, $C_{ps}$. The variations of stagnation coefficient from its non-affected position $\theta = 180^\circ$, the averaged stagnation pressure and base pressure with $\alpha$ for $P/D = 3$ are shown in Figure 2-10.
Figure 2-9 The pressure distribution around two staggered cylinders at various arrangements. $\alpha$, the alignment angle; $C_p$, the pressure coefficient; thin solid line, pressure on upstream cylinder; dash lines, pressure on downstream cylinder.
Figure 2-10 Comparison pressure results as a function of alignment angle for two staggered cylinders between 2-D and 3-D simulations at $P/D = 3$ and Re=$10^3$: (a) the movements of averaged stagnation points where $\Delta \theta (=\theta_s -180^\circ)$ is defined as the difference between the angle of stagnation point ($\theta_s$) and $\theta = 180^\circ$; (b) stagnation pressure; (c) base pressure; □, 3-D upstream cylinder; ○, 2-D upstream cylinder; ×, 3-D downstream cylinder; +, 2-D downstream cylinder; —, 3-D single cylinder; ---, 2-D single cylinder.

(a) **Upstream cylinder**

For Cyl_up, the variation of the alignment angle has little influence on the stagnation
pressure for all $\alpha$ values, as seen from Figure 2-9 for $P/D=1.5$ and 3 and from Figure 2-10 (a). For $P/D=3$ in Figure 2-10 (a), the location of the stagnation of Cyl_up moves less than $1.5^\circ$ to the bottom side for $\alpha \leq 15^\circ$, and $2^\circ$-$5^\circ$ towards the top side for $\alpha \geq 30^\circ$ from the location of $\theta = 180^\circ$. This indicates Cyl_down has very limited influence on the flow around the front face of Cyl_up.

The base pressure changes greatly with the change of both pitch ratio and alignment angle for Cyl_up. For instance in Figure 2-9 (a), the base pressure coefficient is increased from $-0.78$ at $(P/D, \alpha) = (1.5, 0^\circ)$ to $-0.63$ at $(P/D, \alpha) = (1.5, 5^\circ)$ and $-0.62$ at $(P/D, \alpha) = (3, 0^\circ)$. For $P/D=3$ the base pressure coefficient of Cyl_up increases from $-0.62$ to about $-0.44$ as $\alpha$ increases from $0^\circ$ to $10^\circ$, and decreases monotonically afterwards (from $-0.52$ at $\alpha = 15^\circ$ to $-0.91$ at $\alpha = 90^\circ$). The increase of base pressure indicates the weakening of the interactions of the top and bottom shear layers from Cyl_up and the decrease of the base pressure suggests the enhancement of vortex shedding from Cyl_up due to the influence of Cyl_down. The weakening ($\alpha \leq 10^\circ$) and enhancement of vortex shedding ($\alpha \geq 15^\circ$) can also be observed from the rms lift coefficient and vortex shedding frequency ($St$) to be presented later on in Figure 2-12 and Figure 2-16. The weakening of vortex shedding corresponds to small rms lift and low vortex shedding frequency and the enhancement corresponds to large rms lift and high vortex shedding frequency. The base pressure of Cyl_up for $\alpha > 60^\circ$ is close to the base pressure of a single cylinder, indicating the decay of influence from Cyl_down.

(b) *Downstream cylinder*

In contrast, the pressure on Cyl_down is greatly affected by the alignment of the cylinders. It is almost always negative at $\alpha = 0^\circ$ and $\alpha = 5^\circ$ for both $P/D = 1.5$ and 3, where Cyl_down is wholly immersed in a negative pressure zone (a slight positive pressure is found near the stagnation point at $P/D =3 \& \alpha = 5^\circ$). For $\alpha = 0^\circ$ and $\alpha = 5^\circ$, Cyl_down is expected to have a negative drag-thrust force (Zdravkovich 1977), a mean force in the opposite direction of the approaching flow, as the negative pressure in the windward side ($110^\circ \leq \theta \leq 245^\circ$) exceeds the pressure on the base side (approximately, $|\theta| \leq 60^\circ$) of Cyl_down. The stagnation pressure of Cyl_down for $P/D = 3$ increases rapidly from $-0.20$ at $\alpha = 0^\circ$ to about $1.0$ at $\alpha = 10^\circ$ and remains to be around 1.0 for $\alpha \geq 15^\circ$ (refer to Figure 2-10, b). In the meantime, the stagnation point moves rapidly from $\theta = 110^\circ$ at $\alpha = 0^\circ$ to $\theta = 167^\circ$ at $\alpha = 15^\circ$, and then gradually to $\theta = 183^\circ$ at $\alpha = 90^\circ$ (refer to Figure 2-10, a). The base pressure of Cyl_down decreases with larger $\alpha$ from
−0.37 at $\alpha = 0^\circ$ to −0.85 at $\alpha = 45^\circ$, and then stays at about −0.90 for increasing $\alpha$. For $\alpha \geq 60^\circ$, the base pressure on the two cylinders is almost identical to its single cylinder counterpart. It is also observed that the stagnation points of both cylinders are located in the gap sides at $\alpha \geq 60^\circ$ (Figure 2-10, a) and this will induce outward-directed forces on the two cylinders, which will be discussed in details in section 2.3.2.

A significant discontinuous change is found in the pressure distribution on Cyl_down at low alignment angle. It is symmetric about $\theta = 180^\circ$ in the tandem configuration with two peaks appearing at about $\theta = 110^\circ$ and $250^\circ$, respectively. This is because these two locations are attacked by the shear layers from the two sides of Cyl_up. When $\alpha = 5^\circ$, the pressure peak at about $250^\circ$ disappears because this location is immersed in the wake zone of Cyl_up and the value of the other peak increases slightly. The stagnation point rapidly moves towards $\theta = 180^\circ$ location with the increase of alignment angle. The movement of the stagnation point indicates the formation of a biased gap flow between the two cylinders. This gap flow along with the shear layer and vortex interactions are responsible for many results reported here.

(c) Comparison of 2-D and 3-D results

Since available numerical simulations on two staggered cylinder were mainly carried out based on 2-D numerical models (Akbari and Price 2005; Jester and Kallinderis 2003; Lee and Yang 2009; Mittal et al. 1997), we include a brief discussion on the comparison of 2-D and 3-D numerical results. The 3-D results of the variations of the stagnation point, stagnation pressure and base pressure with the alignment angle for two staggered cylinders at $P/D = 3$ and a single cylinder are compared with the 2-D results in Figure 2-10. The 2-D model appears to predict similar results of the variation of stagnation point and stagnation pressure on Cyl_up to those by the 3-D model for $\alpha \geq 45^\circ$ and $\alpha = 0^\circ$. The 2-D model also predicts the trends of the variation of the stagnation point and stagnation pressure of Cyl_down well, but under-predicts the variation angle of the stagnation point and yields a smaller stagnation pressure of Cyl_down for $\alpha < 30^\circ$, which is because the 2-D model predicts the inception of vortex shedding from Cyl_up at a smaller $P/D$ than 3-D model. For example, vortex shedding from Cyl_up is captured by the 2-D model for $P/D = 3.0$ and $\alpha < 30^\circ$ but not by the 3-D model. The prediction of the early inception of vortex shedding from Cyl_up by the 2-D model will be discussed in Section 2.5.2.
Large discrepancies between the 2-D and 3-D results are observed in the base pressure of both cylinders (refer to Figure 2-10c). Coincidentally the 2-D and 3-D models provide similar base pressures of Cyl_{down} for $\alpha \leq 30^\circ$. The 2-D model fails to predict the base pressure of the both cylinders for $\alpha \geq 45^\circ$, which is not surprising because the base pressure of both cylinders for $\alpha \geq 45^\circ$ is close to the base pressure of a single cylinder and the 2-D models generally under-predict the base pressure of a single cylinder at the considered Reynolds number.

![Figure 2-11 Contours of mean drag coefficient on the two staggered cylinders. Left, upstream cylinder; right, downstream cylinder.](image_url)

### 2.3.2 Force coefficient

The influence of the pitch ratio and the alignment angle on the drag coefficient is presented by the contours of the mean drag coefficient in the $P/D-\alpha$ plane as shown in Figure 2-11. Similar to that on the pressure distributions, the arrangement of the two cylinders has much less influence on the force of Cyl_{up} than that of Cyl_{down}. This is because the approaching flow for Cyl_{down} is affected by Cyl_{up}, depending on the location of Cyl_{down}, while the approaching flow for Cyl_{up} is less affected by Cyl_{down}. Both cylinders experience small drag force at low alignment angles, and vice versa. The smallest mean drag coefficient on Cyl_{up} is found to be about 0.82, which is 25% less than that of a single cylinder at $Re = 10^3$ ($\bar{C}_{D} = 1.09$). One interesting feature of the drag forces on Cyl_{up} is that the smallest drag coefficient does not occur in the tandem configuration ($\alpha = 0^\circ$). It occurs at a combination of a pitch ratio of around 2.75 and an alignment angle of around 7.5$^\circ$. It can be seen from pressure distribution that the base pressure of Cyl_{up} approaches to the maximum value at approximately this combination of $P/D$ and $\alpha$, suggesting a less energetic wake behind
Cyl_up. This is because, at small $\alpha$, the existence of Cyl_down disturbs the top shear layers around Cyl_up and weakens the interaction between the top and bottom shear layers around Cyl_up. In contrast, the drag coefficient on Cyl_up is generally enhanced at large $\alpha$. The drag coefficient on Cyl_up is close to but slightly larger than that of an isolated cylinder when $\alpha$ exceeds 45° for all pitch ratios in the present study. This is because, at large $\alpha$, the existence of Cyl_down pushes the top shear layer from Cyl_up closer to the bottom shear layer of Cyl_up, leading to a strong interaction between the shear layers and thus a more energetic wake behind Cyl_up.

It is also observed in Figure 2-11 that the variation gradients of drag force at small $\alpha$ are higher than those at large $\alpha$, indicating dramatic changes in flow condition at small $\alpha$. The drag force on Cyl_down reaches its minimum (negative drag-thrust force) in the tandem configuration at the smallest pitch ratio investigated, where the shielding effect of Cyl_up is the strongest. When the two cylinders are in a tandem arrangement with small $P/D$, Cyl_down is either completely or partially immersed in between the two shear layers from Cyl_up, resulting in small (or even negative) drag coefficients. The mean drag coefficients on Cyl_up are consistently larger than those on the downstream one at small flow approaching angles. It is expected that the magnitude of difference will decrease as $P/D$ increases and will eventually diminish at large pitch ratios. It is observed that both cylinders may experience a smaller drag than that of a single cylinder at certain arrangements and such a feature have been utilized in practical applications such as sports.

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![Figure 2-12 Contour of root-mean-square lift coefficient on two staggered cylinders by 3-D simulations. Left, upstream cylinder; right, downstream cylinders.](image-url)
Figure 2-12 shows the contours of rms lift coefficient on both cylinders for 3-D simulations. The rms lift coefficient on Cyl_up is suppressed at a large region on the $P/D - \alpha$ plane, and it is only at the two corners of the plane ($\alpha \leq 15^\circ$ and $\alpha \geq 75^\circ$ for large pitch ratios) that the rms lift on Cyl_up is close to that of an isolated cylinder (0.20). The maximum rms lift coefficient on Cyl_up occurs when the two cylinders are in the side-by-side arrangement ($\alpha = 90^\circ$) at $P/D = 3$. The rms lift on Cyl_down also peaks at both small and large alignment angles and large pitch ratios. It reaches the maximum when Cyl_down is directly placed behind Cyl_up at the largest pitch ratio 4 in the present study. The rms lift on Cyl_down quadruples the value of an isolated cylinder at $(P/D, \alpha) = (4.0, 0^\circ)$, where the vortices start to be shed from Cyl_up and fully impinge on the downstream one.

Figure 2-13 shows the contours of the mean lift coefficient on the $P/D - \alpha$ plane by 3-D simulations. The lift coefficients on the two cylinders are not zero except when $\alpha = 0$. One prominent feature is the existence of attractive and repulsive lift forces on the two cylinders. The attractive lift force is an inward-directed force (negative on downstream cylinder and positive on the upstream one), indicating the cylinders tend to attract one and another; while the repulsive lift force is an out-directed force (signs are contrary to the attractive force), where the cylinders tend to repel each other (Sumner et al. 2005). Different reasons for the attractive lift (inward-directed) forces have been discovered and reported (Zdravkovich 1977; Ting et al. 1998). In our study, the attractive lift force on Cyl_down is found at $5^\circ \leq \alpha \leq 45^\circ$ for all pitch ratios. The attractive lift force on Cyl_up is only observed in a very small area with small flow approaching angles. It is seen from Figure 2-13 that the attractive lift force on Cyl_down peaks at about $10^\circ$, and the maximum absolute value is 0.34 at $P/D = 3$, corresponding to a high $St$ which will be discussed later on. The obvious attractive lift force on Cyl_down can be explained by referencing the pressure distributions around Cyl_down shown in Figure 2-9 and Figure 2-10, where it is observed that the stagnation point of Cyl_down moves downwards from $\theta = 130^\circ$ at $\alpha = 5^\circ$ to $\theta = 172^\circ$ at $\alpha = 30^\circ$ while the stagnation pressure increases from about 0.37 to about 1.0. The movement of stagnation point leads to a downward drag component. The location of the stagnation point of Cyl_down also suggests that the approaching flow for Cyl_down is directed towards the center-line of the system (i.e. the x-axis as defined in Figure 2-1). This biased flow also contributes to the generation of attractive force by causing a pressure difference in the inner and outer sides of Cyl_down. The pressure distributions are almost symmetric to the approaching
flow direction on Cyl_up for $5^\circ \leq \alpha \leq 30^\circ$. As a result, no obvious attractive force is observed on Cyl_up.

![Figure 2-13 Contour of mean lift coefficient on two staggerd cylinders by 3-D simulations. Left, upstream cylinder; right, downstream cylinders.](image)

The repulsive lift forces (out-directed), on the other hand, are found on both of the two cylinders at large alignment angles ($\alpha \geq 45^\circ$) and are strong at small $P/D$ values (1.5 and 2). The maximum magnitude of the repulsive lift force on Cyl_up is much larger that on the Cyl_down (0.40 compared to 0.26). One of the reasons for the outward repulsive force can be explained, again, by examining the pressure distributions on both cylinders. For instance, in Figure 2-9 at ($P/D, \alpha$) = (1.5, 60°), the stagnation points of the two cylinders are both located in the inner side of the system, creating an outward force on both cylinders. The positive pressure on Cyl_up is much more widely distributed while the base pressure is smaller than those on the downstream one, which are responsible for the difference in magnitude of the repulsive forces. The mean repulsive lift forces observed in this study were reported independently at large subcritical Reynolds number of $3.2 \times 10^4 \sim 7.4 \times 10^4$ (Sumner et al. 2005).

The hydrodynamic forces on two staggered cylinders by 3-D simulations are summarized graphically in Figure 2-14. Emphasis has been put on Cyl_down, as it is affected by the arrangement more than Cyl_up. The solid and the dashed circles stand for the position of the upstream and downstream cylinders, respectively. Negative drag is found for cases with small alignment angles at medium pitch ratios ($\alpha \leq 5^\circ$ and $P/D \leq 3$); for medium angles ($5^\circ \leq \alpha \leq 30^\circ$), the drag forces on Cyl_down is less than that of an isolated cylinder under the same flow condition, but the averaged lift coefficient is
characterized by the attractive force; at larger angles ($\alpha \geq 30^\circ$), the drag forces for two cylinders are less than 15% different from that of a single cylinder, while repulsive force is found in the lift at small pitch ratios ($P/D < 3$) and fades out with the increase of pitch ratio.

![Figure 2-14](image-url) A summary of force characteristics on the downstream cylinder in the two staggered cylinder system.

![Figure 2-15](image-url) FFT Power spectral density of $C_L$. a, $P/D = 2.0$; b, $P/D = 4.0$; 1 and 2 represents upstream and downstream cylinder, respectively.
2.4 Strouhal number

Figure 2-16 Comparison of Strouhal numbers for two staggered cylinders; (a) \(P/D=1.5\); (b) \(P/D=2\); (c) \(P/D=3\); (d) \(P/D=4\). Data of experimental studies are included: ×, Kiya et al. (1980) at Re=15 800; ◊, Sumner et al. (2000) at Re=1350; □, Alam and Sakamoto (2005) at Re=55 000; Δ Hu and Zhou (2008a); ●, present 3-D numerical study; +, present 2-D.

The time history of lift coefficients is analyzed by FFT and the peak frequency of the FFT spectrum is used to determine the \(St\). The FFT spectra of the lift coefficient for \(P/D = 2\) and \(4\) by 3-D simulations are shown in Figure 2-15, where on the left and right are those for the upstream and downstream cylinders, respectively. For \(P/D = 2\), the FFT spectrum is featured by one pronounced peak at \(\alpha = 0^\circ\) and one broadband peak accompanied by many lower peaks at large \(\alpha\). The broadband spectrum indicates the occurrence of very complex flow structures, such as shear layer and vortex interactions which will be presented in Section 2.5 on wake flow regimes. When the pitch ratio is increased to 4, the smaller peaks disappear at large \(\alpha\), suggesting the weakening of interactions between the two cylinders. Another interesting feature is the difference in peak frequencies of the lift force on both cylinders at intermediate \(\alpha\). This indicates the
vortex shedding frequency from the two cylinders is different under wake and proximity interferences.

It is also observed that each FFT spectrum of the lift coefficient has a secondary peak for a medium alignment angle between $15^\circ \leq \alpha \leq 75^\circ$ at $P/D = 2$ (Figure 2-15 a1 & a2). The secondary peak of one cylinder is exactly the primary peak of the other, and thus it is reasonable to conclude that the secondary peak is caused the shear layer or vortex shedding from the other cylinder.

Figure 2-16 shows the variations of the two sets of $St$ with $\alpha$ for different $P$ by both 2-D and 3-D simulations. Of the two sets of $St$ for each flow approaching angle, the smaller one is for Cyl_down and the larger one is for Cyl_up. The experimental data of $St$ reported by Kiya et al. (1980) at $Re=15800$, Sumner et al. (2000) at $Re=1350$, Alam and Sakamoto (2005) at $Re=55000$, and Hu and Zhou (2008a) at $Re = 7000$, are also included in Figure 2-16 for comparison. The $St$ obtained from the reported experiments were determined either by analyzing the frequency of oscillating velocity at certain locations downstream the cylinders (Hu and Zhou 2008a) or by counting and timing individual vortices that are shed from the cylinders (Sumner et al. 2000). In general, the present 3-D numerical results agree reasonably well with the experimental data, despite the differences in flow conditions and statistical method. Although the experiments of Sumner et al. (2000) were carried out at a $Re$ closest to the one considered in the present study, $St$ of Sumner et al. (2000) are greater than the present results and others experimental data at relatively low $P/D$ and/or low $\alpha$, where the flow interactions are vigorous and thus it is hard to count the number of vortices, due to the modulations of the original vortex frequencies and the interactions among the shear layers and vortices. On the other hand, $St$ determined through measuring the velocity at a fixed location may be dependent on the location of the measurement (Sumner 2010). Hu and Zhou (2008a) measured $St$ at six locations ranging from $x/D = 2.5$ to 20. It was found that for the case of $P/D = 2.0$, the $St$ measured at $x/D=2.5$ was more than twice the $St$ measured at $x/D=10$, which was attributed to vortex interactions and evolution. The $St$ determined from lift forces is a direct manifestation of vortex shedding from the cylinders, which will inevitably different from the $St$ determined from quantities measured in the wake flow due to the complex vortex interactions in the wake such as vortex pairing and merging.

The results shown in Figure 2-16 suggest that $St$ of two staggered cylinders is highly
dependent on cylinder configuration. At small $\alpha$, the lift coefficients of both cylinders oscillate with the same frequency for all four spacing ratios (see Figure 2-16 (a) for $\alpha \leq 30^\circ$ and Figure 2-16 (b) for $\alpha \leq 10^\circ$), implying the occurrence of one vortex street. In those cases, the two cylinders behave like one single body. Due to the larger size of the combined cylinders, $St$ is much smaller than that of a single cylinder under the same flow condition.

Two different sets of $St$ were detected at intermediate $P/D$ and intermediate $\alpha$, with one lower than that of an isolated cylinder ($\approx 0.215$) and the other higher. The critical $\alpha$ above which double $St$ values occur decreases with the increasing $P/D$. The difference between the two sets of $St$ tends to diminish with the increasing $P/D$ or $\alpha$, which is obviously due to the weakened wake and proximity interferences. A straightforward explanation of the existence of two sets of $St$ is that the different vortex shedding frequencies from both cylinders contributes to the two distinct values (Zdravkovich 1985; Sumner et al. 2000; Hu and Zhou 2008a). Hu and Zhou (2008a) explained that the Cyl_down imposes a corner effect (as the effect of sharp corners to a square cylinder) to Cyl_up, resulting in a relative high base pressure behind Cyl_up and consequently, a higher vortex shedding frequency than that of an isolated cylinder. While at the same time, Cyl_down is submerged in the wake of Cyl_up with relatively low velocity, which contributes to the lower $St$ associated with Cyl_down.

It is worth mentioning the seemingly abnormal very high $St$ of Cyl_down predicted at $(P/D, \alpha) = (3, 10^\circ)$ shown in Figure 2-16 (c). A similar feature was also found in the experimental study by Sumner et al. (2005) at Re $\sim 10^4$ for $P/D = 2.5$ and $\alpha \approx 7\sim11^\circ$ (of about 0.4). This high $St$ is found to correspond to the maximum attractive lift force.

The $St$ calculated from 2-D simulations is also presented in Figure 2-16 for comparison. The 2-D model also gives two distinct sets of $St$ at $P/D = 1.5, 2$ and 3, but not at $P/D = 4$. However, the values are generally within the envelop formed by the 3-D results, indicating that the 2-D model under-predicts the $St$ on Cyl_up, while over-predicts it on Cyl_down. The comparison also suggests $St$ calculated by the 2-D model at a specific pitch ratio resembles $St$ at a larger pitch ratio predicted by the 3-D model. For instance, $St$ calculated by 2-D model at $P/D = 1.5$ is similar to the $St$ by 3-D model at $P/D = 2$. Since two sets of $St$ are caused by shear layer and vortex interactions in the shared wake, this observation tends to suggest that 2-D model under predicts the flow interference which will be illustrated through flow fields in Section 2.5.
2.5 Wake flow regime

2.5.1 Classification of cross sectional flow fields

The flow regimes of steady flow around two staggered cylinders are examined based on the 3-D simulation results. The flow regimes of steady flow around two staggered cylinders have been classified through experimental studies by Sumner et al. (2000; 2005) and Hu and Zhou (2008a; 2008b). In these studies, flow regimes are classified by analyzing the vortex shedding frequency, force coefficients and the cross section flow field. Four distinct flow structures including two single-street modes (S-I and S-II) and two twin-street modes (T-I and T-II) have been identified by Hu and Zhou (2008a; 2008b) based on two sets of $St$ in the wake, flow topography and their downstream evolution. In the present study, although the similar method as that by Hu and Zhou (2008a) is used to classify the vortex shedding flow, it is anticipated that more underlying physics responsible for the flow regimes can be revealed due to the availability of more detailed flow information in the numerical simulation. It is arguably easier to classify flows in numerical studies compared to experimental studies because the vortex shedding flow field can be easily revealed by visualizing detailed flow fields in any cross-section of the cylinders. The contours of non-dimensional vorticity $\omega$ in the middle cross-section of the cylinders are shown in Figure 2-17 for $P/D = 1.5, 2$ and $4$ at all alignment angles ($P/D = 3$ is given in Figure 2-19 to compare with 2-D simulations), where the vorticity is defined as the curl of velocity field,

$$
\omega_x = \left[ \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right] D U, \quad \omega_y = \left[ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right] D U, \quad \omega_z = \left[ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right] D U
$$

(2-5)

When both $P/D$ and $\alpha$ are small, no vortices are observed in the wake of Cyl_up. The separated shear layers from Cyl_up re-attach to the surface of Cyl_down (shear layer reattachment). Therefore, vortices are shed only from Cyl_down (single wake) and the $St$ for the two cylinders are the same (one $St$). This is essentially vortex shedding mode S-I, as named by Hu and Zhou (2008a). The wake flow for case of $P/D = 1.5$ ($\alpha \leq 30^\circ$) falls in the S-I regime. As $P/D$ increases, the highest alignment angle $\alpha$, below which mode S-I exists, decreases. It is reduced to $5^\circ$ for $P/D = 3$ and when $P/D$ increases to 4, no S-I mode is observed and vortex shedding occurs from both cylinders at all alignment angles investigated in this study.

Vortex shedding from both cylinders can be detected with either slightly increased
$P/D$ or $\alpha$ from Regime S-I, but vorticity from the pair of cylinders fully merge together to a single asymmetric wake. Thus, there are usually two distinct $St$ for the two cylinders, as revealed by the lift forces, but only one $St$ in the wake. This flow feature is named as regime S-II. For $P/D = 1.5$ and 2, the vortices that are shed from the bottom cylinder are weak and dissipated quickly due to the strong influence from the bottom shear layer of the top cylinder. The shear layer from the bottom side of the top cylinder is strong due to the jet-flow through the gap between the cylinders. The vortex shedding mechanism in regime S-II for $P/D = 3$ and 4 at large alignment angles is generally different from that for $P/D = 1.5$ and 2. For $P/D = 3$ and 4, the vortices that are shed from Cyl\_up attack Cyl\_down and merge with the vortices shed from Cyl\_down, forming a single wake behind the cylinder pairs.

The wake is characterized by two Kármán vortex streets that interact with each other strongly as $P$ or $\alpha$ are further increased. The $St$ values for these two Kármán vortex streets are different and maintain so in the wake, leading to flow regime T-I. The interactions between the shear layers in regime T-I are not as significant as those observed in regime S-I and S-II. This is because Cyl\_down is further away from the wake zone of Cyl\_up; however, strong interactions among the vortices from the two streets are still observed in the wake, including vortex pairing.

It is observed from the flow fields that S-II and T-I are featured with biased vortex shedding, where vortex cores shed from Cyl\_down are considerably larger than those from Cyl\_up and the gap flow is directed downwards to the wake of Cyl\_up. The relative smaller vortex cores from Cyl\_up are due to the confined space (built-up by Cyl\_down itself and its shear layer) in its wake which restricts the growth of vortex cores and also because of the generation of energetic wake flow as observed from force coefficients. As a result, small vortex from Cyl\_up takes a shorter period of time to develop and shed than large vortex from Cyl\_down. This explains the existence of two sets of $St$. Depending on how close the two cylinders are, the shed vortex (with different frequency) from the two cylinders may or may not merge together, leading to regime S-II or T-I, respectively. The biased flow is not capable of changing its inclined direction in the staggered arrangement in regime S-II and T-I; however, at the side-by-side configuration, gap flow intermittently flips over. Whichever cylinder the flow inclines to, its wake is half enclosed and thus the vortex shedding is restrained; as a result the cylinder experiences higher $St$ than the other. It is still unclear when and how the gap
flow in the side-by-side arrangement alters its biased direction and this remains to be an interesting topic to study. It is worth mentioning that the existence of a stable deflection regime has been found by Williamson (1985) even in a perfect symmetric system. In such cases, the two side-by-side cylinders behave differently and lead to different statistic data, such as $St$.

![Figure 2-17 Instantaneous vorticity contours in the mid-section of the cylinders illustrating the different flow regimes observed in the flow around two staggered circular cylinders; Left, $P/D = 1.5$; middle, 2; right, 4; Contours cutoff level $= 0.2$; The background dashed square is $2D$ in size.](image)

Furthermore, at the side-by-side or nearly side-by-side arrangements and at large pitch ratios ($P/D \geq 3$), the flow field is characterized by two vortex streets with weak or no interactions, and is named regime T-II. In this flow regime, the vortex shedding from one cylinder is similar to that from the other, and $St$ of each cylinder is close to that of
an isolated cylinder. The flow interference in T-II is the weakest in all the regimes identified above due to the weak wake interference or the proximity interference at large \( P/D \) and \( \alpha \). Regime T-II is among the first studied wake pattern around two cylinders (Williamson 1985). Because the two vortex shedding processes from both cylinders are found to be either in-phase or in anti-phase with each other, regime T-II was named synchronized vortex shedding or anti- and in-phase vortex shedding in previous studies (Bearman and Wadcock 1973; Williamson 1985).

It should be noted that above flow classifications are not possible without quantitative measurement of velocity singles in the wake, especially at around boarder lines where the flow regimes cannot simply be classified based on flow structures and FFT lift spectrum only. For instance no obvious differences can be detected between the flows with \((P/D, \alpha) = (2, 45^\circ)\) and \((2, 60^\circ)\) based on qualitative observations on the flow field and the spectrum of lift coefficient. Their classifications of flow regimes of S-II and T-I respectively are achieved by FFT analysis of flow velocity samples in the wake of the cylinders. In this case, we sampled horizontal velocity component \( U_x \) at 11 spatial points along a line of \( x = 10D \) \((-4.5D \leq y \leq -4.5D)\) downstream of the cylinder system (see Figure 2-1) in the middle plane perpendicular to the cylinder and conducted the FFT analysis of the velocity signals. The results of the FFT analysis are listed in Table 2-5. It is seen that \( St \) remains a constant across the wake for the case of \((P/D, \alpha) = (2, 45^\circ)\), suggesting a single wake and thus being classified as S-II, while two distinct \( St \) values are detected in the wake of the cylinders for \((P/D, \alpha) = (2, 60^\circ)\), indicating the interactions between two wakes, and thus being classified as T-I.

The flow regimes identified in this study are mapped out on the \( P/D - \alpha \) plane as shown in Figure 2-18, which is similar to figure 14 in Hu and Zhou (2008a), despite the differences in Reynolds number in the two studies. The case of \((P/D, \alpha) = (1.5, 15^\circ) \& (1.5, 30^\circ)\) was classified as S-I because \( St \) for both cylinders are the same, while the case of \((P/D, \alpha) = (4, 0^\circ)\) was classified as S-II because vortices are found to be shed from both cylinders in the present study. In general S-I occurs at small alignment angles. The boundary value of \( \alpha \) between S-I and S-II regimes decreases with the increasing \( P/D \). Similarly, the boundary value of \( \alpha \) between S-II and T-I regimes also decreases with the increasing \( P/D \). The T-II regime only occurs at large \( P/D \) and large \( \alpha \).
Table 2-5 Variation of Str at 10D downstream the staggered cylinder system for \( P/D = 2 \)

![Table 2-5 Variation of Str at 10D downstream the staggered cylinder system for \( P/D = 2 \)](image)

Figure 2-18 Map of flow classifications around two staggered circular cylinders in steady cross flow at Re = 10^3; S-I, single vortex street with vortex only shed from downstream cylinders; S-II, single street with vortex shed from both cylinders; T-I, twin streets with strong interactions; T-II, twin streets with weak interactions; ●, the cases investigated in the present study.

2.5.2 3-D effects on the flow fields

In all the reviewed studies on two staggered cylinders, the flow field classifications were based on sectional flow visualization, pressure distributions and/or velocity signals. Surprisingly, the 3-D effects on the flow field have not been investigated.

The flow fields predicted by 2-D and 3-D models at \( P/D = 3 \) are qualitatively compared in Figure 2-19, since this pitch ratio covers all flow regimes as discussed in Figure 2-18. Generally speaking, the vortices in the 2-D model are more energetic and regular than those in 3-D model, and remain so further downstream of the wake. This is mainly due to the absence of disruptions to the two-dimensional flow structures from three-dimensionality, such as, the energy transferring to the streamwise vortices and vortex dislocations. On the other hand, the shared wake predicted by the 3-D model comprises of many small scale vortex structures. The flow fields calculated by the 2-D model for all other cases show similar features and thus will not be shown here. In Figure 2-19, the 2-D model fails to predict the shear layer reattachment regime at \( \alpha = 0^\circ \)
and 5° (S-I) revealed by the 3-D model, which is consistent with the drag force feature between the two models. Strong vortices shed from Cyl_up are predicted by the 2-D model for these two cases. Since the vortex interactions predicted by the 2-D model are also less obvious for α = 60° and 75° and the Strouhal numbers for the two cylinders are equivalent, the flow is classified as regime T-II in contrast to the flow regime T-I predicted by the 3-D model. It appears to be clear that it is inappropriate to classify flow around two staggered cylinders based on 2-D simulation results.

![Image: Comparison of the instantaneous vorticity contours in the cross-section of the cylinders between 3-D and 2-D numerical simulations.](image)

Figure 2-19 Comparison of the instantaneous vorticity contours in the cross-section of the cylinders between 3-D and 2-D numerical simulations. Contours cutoff level = ±0.2; The background dashed square is 2D in size.

To quantify the extra energy dissipation of the 3-D flow fields observed in Figure 2-19, the enstrophy of flow around the two-cylinder system is computed. The enstrophy,
which represents a measure of kinematic energy that corresponds to the dissipation effect in the flow field, is determined as the integral of the squared vorticity \( \omega_i \) \( (\omega_x, \omega_y, \omega_z) \) given a velocity field \( \vec{U} \) through a selected computational domain \( \Omega \) (Papaioannou et al. 2006),

\[
e(t) = \int_{\Omega} \omega^2_i(x, t) dV = \int_{\Omega} ||\nabla \times \vec{U}||^2 dV
\]  

(2-6)

Therefore, the enstrophy is time-dependent scalar and is calculated by squaring and integrating the magnitude of the vorticity. In the 3-D simulations, the vorticity is a three component vector, but has only one non-zero component in the \( x-y \) plane (\( \omega_z \)) in 2-D simulations. Thus, the primary and secondary enstrophy are introduced to distinguish the enstrophy based on different components of the vorticity field. The primary component of the enstrophy is calculated through spanwise (\( z \)) vorticity,

\[
e_z(t) = \int_{\Omega} \omega_z^2(x, t) dV
\]  

(2-7)

Similarly, the secondary enstrophy is calculated by summing the streamwise (\( x \)) and lateral (\( y \)) components of the vorticity, i.e.

\[
e_{xy}(t) = \int_{\Omega} \left[ \omega_x^2(x, t) + \omega_y^2(x, t) \right] dV
\]  

(2-8)

Since the secondary enstrophy only exists in 3-D simulations, it is an alternative way to quantify the three-dimensionality of the flow. These definitions make the enstrophy shown in equation (5-2) as the total enstrophy. It is readily seen that the total enstrophy in 2-D simulations is equivalent to the primary enstrophy. The results of calculated enstrophy for \( P/D = 3 \) are presented in Figure 2-20, along with the corresponding data for the single cylinder case in the same flow condition. The enstrophy shown in Figure 2-20 is normalized by the inlet velocity and height of the computational domain, formulated as,

\[
e' = \frac{e}{(U^2H)}
\]  

(2-9)
Figure 2-20 Variation of total enstrophy (a), primary enstrophy (b), and enstrophy proportion (c) as a function of alignment angle for $P/D = 3$; □, 3-D; +, 2-D; —, 3-D single cylinder; ---, 2-D single cylinder; ●, $\varepsilon_z/\varepsilon$; ♦, $\varepsilon_{xy}/\varepsilon$.

It is observed that the total enstrophy is very low at small alignment angles. This is the reason why the wake seems to be more robust at the tandem arrangement. The total enstrophy shows a trend of increase with increasing $\alpha$, indicating the increase of fluid
rotation and energy dissipation; and it reaches about two times of the enstrophy of the single cylinder case at regime T-I and T-II, due to the vortex shedding from both bodies. It is not surprising to see that the deviations between 2-D and 3-D models are small, which are within 15% difference at $\alpha > 10^\circ$; however, it is interesting to see the 2-D model over-predicts the total enstrophy when $\alpha \leq 5^\circ$ (regimes S-I). The over prediction of enstrophy by the 2-D model at low alignment angle is caused by the early inception of vortex shedding in the gap, which was also discussed by Papaioannou et al. (2006) for two tandem cylinders.

Figure 2-20(b) shows that the primary enstrophy of the two-cylinder system is restrained to the value of a single cylinder at tandem configuration; and again, it grows as the Cyl\_down moves away from the wake and roughly doubles the enstrophy of the single cylinder case after $\alpha \geq 45^\circ$. The primary enstrophy of the 3-D simulations is significantly lower than that of the 2-D simulations, and the discrepancy further increases at large alignment angles. These large deviations of the primary enstrophy are obviously attributed to the kinetic energy in the $z$-direction. This quantitative analysis is consistent with the qualitative observations of the differences in flow structures predicted by the 2-D and 3-D models shown in Figure 2-19, where much disturbed flow fields is seen in 3-D simulations.

For flow around a single cylinder at $10^3$, the primary component of the enstrophy only takes up about 60% the energy dissipation, so the three-dimensionality contributes to almost 40% of the total enstrophy. Figure 2-20(c) illustrates the composition of enstrophy of flow around the two-cylinder system. It is observed that three-dimensionality of the flow is significantly suppressed at low $\alpha \leq 30^\circ$ in regimes S-I and S-II, indicating a more robust 2-D wake could be achieved by simply extending the length of a structure in the direction of the flow. At the tandem arrangement, more than three quarters of the enstrophy is primary enstrophy. Although the proportion of secondary enstrophy in the 3-D simulations grows significantly with the larger alignment angles in two single-wake regimes, S-I and S-II, it stays flat in regime T-I, nearly equivalent to the value of the single cylinder case, followed by a slight raise at $\alpha = 90^\circ$ (regime T-II). Therefore, the flow interaction around multiple structures does not necessarily increase the three-dimensionality.

We further investigate the flow field by comparing the stretching of the vorticity with the diffusion of the vorticity, as expressed in vorticity transport equations,
\[
\frac{D\omega}{Dt} = (\omega \cdot \nabla)U + \nabla \times \omega
\]  

(2-10)

The first term on the right hand side of Equation (5-6) represents vortex stretching, which is absent in 2-D flows, while the second term denotes vortex diffusion, which only occurs in viscous flows, since the coefficient of vortex diffusion is the kinematic viscosity. This study calculates the competition between the terms of vortex stretching and vortex diffusion by comparing their integration over a selected computational domain covering the cylinders and the wake region. To facilitate the comparison, the resulted competition coefficient is normalized by the height of computational domain and the diameter of the cylinders, formulated as,

\[
C'_{\omega} = \frac{\int (\omega \cdot \nabla)U dV}{\int |\nabla^2 \omega| dV / HD^2}
\]  

(2-11)

Figure 2-21 illustrates the variations of time-averaged \(C'_{\omega}\) with alignment angle at three pitch distances, along with that of the isolated cylinder at Re = 10^3. It is observed that \(C'_{\omega}\) generally increases with \(\alpha\) and is impervious to the pitch distance until a critical alignment angle is reached. It is observed that the magnitudes of \(C'_{\omega}\) do not change with gap distance for \(\alpha \leq 45^\circ\). This suggests that \(\alpha\) is a dominant factor for vortex stretching for \(\alpha \leq 45^\circ\). This is consistent with the observation in regimes S-I and S-II, where vortex shedding from the upstream cylinder is significantly constrained at low \(\alpha\), regardless of pitch distances. It is seen that \(C'_{\omega}\) does not increase with \(\alpha\) significantly for \(\alpha \geq 75^\circ\), suggesting that the pitch ratio becomes a dominant factor for vortex stretching for \(\alpha \geq 75^\circ\). 

![Figure 2-21 Variation of the competition between terms of vortex stretching and vortex diffusion in the vorticity transport equations; ◊, P/D = 2; □, P/D = 3; Δ P/D = 4; ―, single cylinder.](image-url)
Bloor (1964) found that the wake behind a single cylinder becomes turbulent at \( \text{Re} \geq 400 \) and the shear layer behind a single cylinder becomes turbulent at about \( x/D = 2.0 \) for \( \text{Re}=1.35 \times 10^5 \). The transition to turbulence in the shear layers are observed by sampling velocity singles at a number of discrete points in the downstream wake of both a single cylinder and two cylinders in a tandem arrangement with \( P/D = 3 \) in this study, as shown in Figure 2-22. For the single cylinder case, small scale oscillations are observed for \( x/D \geq 1.6 \), although high frequency velocity is more obvious at \( x/D \geq 2.0 \). These small scale, high frequency velocity oscillations are thought to be a typical signature of turbulence. With the existence of an additional cylinder, it is seen from Figure 2-22 that the velocity signals sampled behind the tandem cylinders appear to be smoother than those sampled at the corresponding locations downstream of the single cylinder. This tends to suggest that the transition to turbulence for the tandem cylinder at \( P/D = 3 \) occurs further downstream, compared with the single cylinder case. It is speculated that the delay in the transition to turbulence behind the tandem cylinders is due to the weak three dimensionality of the flow in the cylinder wakes. It is recognised however that the results shown in Figure 2-22 are less conclusive and are justified for a
separate investigation.

2.5.3 Variation of the flow in the spanwise direction

The variation of the flow fields in the spanwise direction of the cylinder is further investigated by showing the second negative eigenvalue $\lambda_2$ of the tensor $\Psi^2 + \Omega^2$. Here $\Psi$ and $\Omega$ are the symmetric and the anti-symmetric parts of the velocity-gradient tensor, respectively. It has been demonstrated that the second eigenvalue $\lambda_2$ can be used to accurately identify the location of the vortex cores, because it captures the minimum pressure in a plane perpendicular to the vortex axis at both high and low Reynolds numbers (Jeong and Hussain 1995).

Shown in Figure 2-23 are the iso-surfaces of non-dimensional $\lambda_2 = -1$ under the influence of alignment angles at $P/D = 3$. The vertical tubes in the iso-surfaces of $\lambda_2$ generally indicate the locations where vortices in the $z$-direction ($\omega_z$) dominate, while the horizontal rib-like tubes represent the locations where the vortices in the directions perpendicular to the cylinders ($\omega_x$ or $\omega_y$) dominate.

In all arrangements, the vertical vortex structures are highly inclined in some cases, suggesting the high level of three dimensionality of the flow around the cylinders. One such example is seen at $\alpha = 15^\circ$ with greatly distorted wake. Another feature in the flow field is that the intensity of rib-like vortex flow structures increases with the increase in the alignment angle. When one cylinder is immersed in the wake of the other, the two cylinders behave like an elongated body in the streamwise direction and the shielding effects discourage the generation of three-dimensionality, as quantified in the enstrophy calculation. This is especially evident in the gap between the two cylinders at low alignment angles, where only a few rib-like vortices are seen, for instance, at $\alpha = 0^\circ$, $5^\circ$ and $10^\circ$.

At high alignment angles, the rib-like vortices dominate the wake flow, leading to the increase of enstrophy of the flow fields. At high alignment angle in the shared wake, these spanwise flow structures cut the vertical tubes that are seen at $\alpha = 0^\circ$ and $5^\circ$ into pieces and therefore can hardly be found at $\alpha = 75^\circ$ and $90^\circ$. As a result, the primary enstrophy accounts for a major part of the total enstrophy in the flow field. It is reasonable to conclude based on 3-D flow field observations that the presence of a downstream cylinder not only weakens the 3-D flow structures in the gap region between the two cylinders, but also in the shared wake of the two cylinders.
A single wake is formed at low incident angles ($\alpha = 0^\circ$ & $\alpha = 5^\circ$), where the separated shear layers from Cyl_up reattach onto the surface of the downstream one, instead of forming vortices within the gap. It can be seen in Figure 2-23 that very few vortices are located in the gap at $\alpha = 0^\circ$ and $5^\circ$. This is similar to the flow reattachment regime at intermediate pitch ratios for two tandem cylinders. At $\alpha = 10^\circ$, wake vortex shedding from Cyl_up can be identified. The vortices that are shed from Cyl_up are very weak and they are attracted towards the inner side of Cyl_down and then merge with the vortices from the inner side of the downstream, forming a single wake in regime S-II. The three-dimensional flow feature at $\alpha = 15^\circ$ is similar to that at $\alpha = 10^\circ$ but with stronger vortices being shed from Cyl_up.

Two wakes can be clearly seen for alignment angles larger than $45^\circ$ inclusive, where the vorticity shed from both cylinders are convected in the downstream direction for a
considerable distance forming a flow regime T-I. Strong interaction between the two vortex streets in the wake leads to a very chaotic flow pattern in flow regime T-I. At larger alignment angles (α=75° and 90°) the flow in the wake of each cylinder is very similar to that in the wake of an isolated single cylinder. At α = 90°, the vortex shedding processes from the two cylinders are either in-phase or in anti-phase with each other, forming the so-called synchronized vortex shedding (Sumner et al. 2000).

The variation of the flow in the spanwise direction of the cylinders is further investigated by analyzing the variation of pressure coefficient along the spanwise direction. The time evolution of the distribution of the base pressure (pressure exerted on aft end of the body) on Cyl_down along the spanwise direction is shown in Figure 2-24, for three selected alignment angles of α = 5°, 30° and 75°, representing flow regimes S-I, S-II and T-I, respectively (regime T-I and T-II are too similar to be distinguished from each other based on pressure). In general, the base pressure decreases significantly with increasing α. This decrease in base pressure contributes to the increase of drag coefficient. It is also observed from the contours of the pressure coefficient that the spanwise flow features become more obvious when Cyl_down moves out from the wake of Cyl_up. At α = 5°, the pressure colour ribbons are generally intact and parallel to the z-axis, for instance, between 320 ≤ Ut/D ≤ 340, although the pressure colour ribbons are less intact at around Ut/D = 310, 350–390. A single colour ribbon (to cylinder axis) indicates strong two-dimensionality of the flow.

Figure 2-24 Time histories of the base pressure coefficients on the downstream cylinders in spanwise direction at P/D = 3 for three selected alignment angles.

(a) α = 5°

(b) α = 30°

(c) α = 75°
The discontinuity of pressure colour ribbons is more prevailing at $\alpha = 30^\circ$, where only occasionally, parallel pressure colour ribbons are observed. At $\alpha = 75^\circ$, pressure ribbons are broken into small pieces, indicating the dominance of fine flow structures and weakening of organized flow modes. The change in base pressure distribution is a manifestation of lessening of the shielding effect and wake interference with the increase of alignment angle.

The variation of the flow in the spanwise direction is quantified by measuring the spanwise standard deviation of the base pressure at an instant $C_{pb}^{dev}(t)$ for the two cylinders, which is defined as,

$$C_{pb}^{dev}(t) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (C_{pb}(z_i, t) - \overline{C_{pb}(t)})^2} \quad (2-12)$$

where $\{C_{pb}(z_1, t), C_{pb}(z_2, t), \ldots, C_{pb}(z_N, t)\}$ are the observed values of base pressure at an instant along the spanwise direction ($z$), while the denominator $N$ stands for the size of the sampling points and a total of 100 points were sampled at each of the base line along the spanwise length of the two cylinders; $\overline{C_{pb}(t)}$ is the instantaneous mean value of these observations, which can be formulated as,

$$\frac{1}{N} \sum_{i=1}^{N} C_{pb}(z_i, t) \quad (2-13)$$

Then averaging in the time domain of $C_{pb}^{dev}(t)$ yields $C_{pb}^{dev}$. The method measures the rms value of base pressure in the spanwise direction and then averages it in time, thus can be regarded as an indicator of spanwise variation. The results for the case of $P/D = 3$ are presented in Figure 2-25, including both up- and downstream cylinders, as well as those for a single cylinder for comparison. The flow regimes S-I, S-II, T-I and T-II are also marked from left to right for this case. The pressure fluctuation on Cyl_up are considerably smaller than that of a single cylinder for $\alpha \leq 60^\circ$, while it is much larger on Cyl_down than the single cylinder case for $\alpha \geq 15^\circ$. For the same reason as explained in drag forces, the minimum fluctuations are found at a slight inclined arrangement of around $\alpha = 10^\circ$ instead of $\alpha = 0^\circ$ for both cylinders (0.012 and 0.051, respectively). In regime S-II, a leap of pressure fluctuations is observed because of inception of vortex shedding from Cyl_up. The fluctuations of pressure on Cyl_down stay nearly flat at the end of regime S-II and T-I because the interaction has reached to its maximum; on the
contrary, Cyl\_up experiences a steady increase from 0.029 at $\alpha = 30^\circ$ as the three-dimensionality increases in the gap region. At $\alpha = 90^\circ$, as anticipated, the fluctuations are close for both cylinders of about 0.08, which is slightly higher than the value for the isolated cylinder (0.06) due to the flow interference from the two cylinders.

Figure 2-25 Mean base pressure fluctuations for two staggered cylinders of $P/D = 3$ as a function of alignment angle at $Re=10^3$; □, upstream cylinder; Δ, downstream cylinder; ---, single cylinder.

2.6 Conclusions

Steady uniform flow around two staggered cylinders is simulated numerically at a low subcritical Reynolds number of $10^3$. The flow through and after the two-cylinder system are analyzed for pitch ratios ($P/D$) of 1.5, 2, 3 and 4 with the flow incident angle ($\alpha$) between $0^\circ$ and $90^\circ$. The main conclusions are summarized below.

1. The pressure distribution on each of the two cylinders changes significantly with the change of arrangement, especially when $P/D \leq 3$ and $\alpha \leq 60^\circ$. The movements of stagnation points and variations in pressure distribution around the cylinder surface are the manifestations of the vigorous interference in the flow field under the presence of another cylinder.

2. Due to the shear layer and vortex interactions, both cylinders experience smaller drag force at low alignment angles than that of a single cylinder, and vice versa at high alignment angles. Negative drag is found on the downstream cylinder for cases with small alignment angles and medium pitch ratios ($\alpha \leq 5^\circ$ and $P/D \leq 3$). Because of the biased flow field and the gap flow, the mean lift coefficient is characterized by the attractive force at medium alignment angles ($5^\circ \leq \alpha \leq 30^\circ$) and by repulsive force at large alignment angles ($\alpha \geq 30^\circ$). All these force features becomes less
obvious with the increase of pitch ratio.

3. Each of the two cylinders exhibits an individual vortex shedding frequency for intermediate pitch ratios and alignment angles. The $St$ of the upstream cylinder is higher because the limited space in the wake restricts the growth of large vortex cores. Instead only small scale vortices are shed from upstream cylinder, which takes a short period of time to develop. On the other hand, there is less limited space in the wake of the downstream cylinder for large vortex to grow in longer time, which forms a biased flow deflecting to the upstream cylinder.

4. Four distinct vortex shedding regimes are identified based on numerical flow visualization, FFT analysis of lift forces and FFT analysis of velocity sampled from the wake of the cylinders. It is demonstrated through numerical examples that accurate classification of vortex shedding regimes behind two staggered cylinders can only be achieved by 3-D simulations (rather than 2-D simulations) and also by the combination of the flow visualization and the FFT analyses of lift forces and velocity.

5. It is found that the flow approaching angle has significant effect on the three-dimensionality of the flow, which applies not only to the gap region between the two cylinders, but also to the shared wake of both cylinders. This is qualitatively shown by the 3-D flow fields and quantitatively demonstrated by the enstrophy and spanwise pressure fluctuations. Interestingly and probably surprisingly, active wake interaction at medium distances around two-staggered cylinders, which brings much disturbed flow field, does not obviously increase the three-dimensionality of the flow.

Acknowledgement

This work was supported by Australian Research Council Discovery Grant (Project ID: DP110105171) and by iVEC through the use of advanced computing resources (Epic and Magnus supercomputers) located at iVEC@Murdoch. The first author would like to acknowledge the support of the Australian Government and the University of Western Australia by providing SIRF and UIS scholarships for a doctoral degree.

REFERENCES


Chapter 3

Classification of wake flow patterns around four cylinders in a square arrangement in steady flow

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Abstract: Steady uniform flow around a four-cylinder array in a square arrangement is numerically simulated at cylinder Reynolds numbers $Re$ ranging from 50 to 300 with an interval of 10 and center-to-center spacing to cylinder diameter ratios from 1.7 to 4.5. Wake characteristics behind cylinders and hydrodynamic forces on the cylinders are investigated by carrying out a large number of numerical tests. It is found that vortex shedding around cylinders, wake characteristics and hydrodynamic forces on the cylinders are dependent on Reynolds numbers and spacing ratios. A total of seven flow regimes are identified and mapped on the plane of Reynolds number and space ratio. Each of the seven flow regimes, which are referred to as Pattern A to Pattern G, has distinguished flow features, resulting from the interactions among the shear layers, Kármán vortex streets behind the cylinders and vortices shed around cylinders. Some of the flow features around the four cylinders, such as the single bluff-body vortex shedding, binary vortex shedding, in-phase vortex shedding, biased vortex shedding,

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and vortex co-shedding & anti-phase synchronization, share similarities with flow features exhibited by flow around two side-by-side cylinders and flow around two tandem cylinders. Some of the flow features, such as triple vortex streets and the in-phase escape of vortices at the onset of vortex shedding from upstream cylinders found in the in-phase vortex shedding regime, are unique to the four-cylinder system and have not been reported previously.

3.1 Introductions

Despite extensive studies in the past decades, vortex shedding patterns induced by uniform flow around cylinders are still the focus of many studies nowadays due to its rich physics and relevance to engineering applications. Comprehensive reviews, to name a few, can be found in Williamson (1996), Sumer and Fredsøe (1997) and Zdravkovich (1997, 2003) for flow past a single cylinder and in Sumner (2010) for flow around a pair of circular cylinders in side-by-side, tandem and staggered configurations.

When two or more circular cylinders are close to each other, the proximity interference in a side-by-side arrangement and the wake interference in a tandem configuration lead to much more complex flow features than those in the wake of a single cylinder (Zdravkovich, 1987). Flow around four bluff bodies in a square configuration exhibits characteristics of both proximity and wake interferences. The present study is motivated by the combination of the complicity of the flow and the wide engineering applications of multiple cylindrical structures that are commonly used in offshore oil and gas engineering.

In addition to Reynolds numbers, the vortex dynamics in the wake of a cylinder array are greatly influenced by the spacing between the cylinders and the alignment angle of the cylinder array to the flow direction. The non-dimensional distance, spacing ratio $P^*$, between any two adjacent cylinders is defined as the ratio of the center-to-center distance, $L$, to the cylinder diameter, $D$, i.e. $P^* = L/D$. The alignment angle is the angle between the direction of incoming velocity of uniform flow and the arrangement of multiple structures.

For two cylinders of equal diameter in a side-by-side configuration, three main flow patterns due to proximity interference were found based on the spacing ratio (Sumner, 2010). They are (i) single-bluff-body behavior at small $P^*$, where the two cylinders are close enough to each other to act as a single structure; (ii) a biased flow pattern at
intermediate $P^*$, where the gap flow biased towards one of the two cylinders, resulting in asymmetric flow field; and (iii) parallel vortex streets at large $P^*$, where the flow field is symmetry with two Kármán vortex streets in the wake of both cylinders.

<table>
<thead>
<tr>
<th>Researchers</th>
<th>$Re$</th>
<th>$P^* = L/D$</th>
<th>Method</th>
<th>Alignment</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sayers (1988)</td>
<td>$3 \times 10^4$</td>
<td>1.1–5</td>
<td>Exp</td>
<td>0°–45°</td>
<td>$C_D, C_L$</td>
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<tr>
<td>Sayers (1990)</td>
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<td>1.5–5</td>
<td>Exp</td>
<td>0°–45°</td>
<td>$St$</td>
</tr>
<tr>
<td>Lam and Lo (1992)</td>
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<td>1.28–5.96</td>
<td>Exp</td>
<td>0°–45°</td>
<td>$St, FL$</td>
</tr>
<tr>
<td>Lam and Fang (1995)</td>
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<td>1.26–5.80</td>
<td>Exp</td>
<td>0°, 15°, 45°</td>
<td>$C_p, C_D, C_L$</td>
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<td>3.0–5.0</td>
<td>Num</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>1.69–3.83</td>
<td>Exp</td>
<td>0°–45°</td>
<td>$C_D, C_L, St$</td>
</tr>
<tr>
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<td>4</td>
<td>Exp</td>
<td>0°–45°</td>
<td>FL</td>
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<td>0°</td>
<td>FL</td>
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<td>0°</td>
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</tr>
<tr>
<td>Han (2013)</td>
<td>200</td>
<td>1.5–4.0</td>
<td>Num</td>
<td>0°, 45°</td>
<td>$C_D, C_L, FL$</td>
</tr>
</tbody>
</table>

Table 3-1 Selected studies of four circular cylinders in cross-flow. CBEM = Cell boundary element method; LIF = Laser-induced fluorescence; PIV= particle image velocimetry; Exp = Experimental; Num = Numerical; FL = flow visualization.

For two cylinders of equal diameter in a tandem configuration, eight flow patterns in total were identified by Igarashi (1981, Igarashi, 1984) depending on the spacing ratio and Reynolds number. These eight flow patterns were later grouped into three (Xu and Zhou, 2004, Zdravkovich, 1987, Sumner, 2010), namely, (i) single bluff-body behavior or extended-body regime at small $P^*$; (ii) shear layer reattachment or reattachment regime at intermediate $P^*$, where the separated shear layers from the upstream cylinder reattach to the downstream cylinder; and (iii) Kármán vortex shedding from each cylinder or co-shedding regime at large $P^*$.

Studies on flow around four and more cylinders have not been well documented in literature. Some available experimental and numerical investigations of four cylinders in fluid flow are summarized in Table 3-1. The angle of alignment in Table 3-1 is the angle between the flow and the side boundary of the square configuration of the cylinders and varies from 0° to 45°. It is noted from Table 3-1 that all experimental studies were conducted at Reynolds numbers of around $10^3$–$10^4$ while most of the numerical studies were carried out at a relatively small Reynolds number of $Re=200$. Three distinct flow patterns were detected (Lam et al., 2003b) with the change of the spacing ratios and four groups of flow features were captured by varying the alignment angle (Lam et al., 2003a) for $Re=200$. The flow around four cylinders in a square arrangement is also influenced by the Reynolds number especially at low Reynolds number regime. This paper aims for a detailed study of flow patterns around four cylinders.
cylinders in the in-line square configuration and the influences of $Re$ and $P^*$ on flow features. The present study is based on two-dimensional (2D) numerical simulations at Reynolds numbers up to 300.

3.2 Numerical method

3.2.1 Problem definition

A schematic diagram of the problem under consideration is shown in Figure 3-1 (a). Four cylinders of the same diameter $D$ are placed in steady uniform flow in a square arrangement with its two sides being aligned in the direction of the incoming flow. In the discussion, the four cylinders are labeled as number 1 to 4 as shown in Figure 3-1. The incoming uniform steady flow velocity is $U$. Numerical results for $50 \leq Re \leq 300$ (based on the cylinder diameter) with an interval of 10, and $1.7 \leq P^* \leq 4.5$ are presented.

3.2.2 Numerical model

Steady uniform flow around the four-cylinder system is simulated by solving the incompressible Navier–Stokes (NS) equations. The vector form of 2D NS equations for incompressible flow in the Cartesian coordinate system can be expressed as

$$\frac{\partial \vec{U}}{\partial t} + \nabla \cdot (\vec{U} \vec{U}) - \nu \nabla^2 \vec{U} = -\frac{\nabla p}{\rho} \quad (3-1)$$

$$\nabla \cdot \vec{U} = 0 \quad (3-2)$$

where the velocity vector $\vec{U}$ has two components $U_x$ and $U_y$ in the $x$- and $y$-directions respectively, $\rho$ is the density of the fluid, $\nu$ the fluid kinematic viscosity, $t$ the time, and $p$ is the pressure. The NS equations are solved using the Open source Field Operation and Manipulation (OpenFOAM®) C++ libraries, which is a free source CFD package developed by OpenCFD Ltd. The finite volume method is used in the solver and the pressure-velocity coupling is dealt by the method of Pressure Implicit with Splitting of Operators (PISO). The convection terms are discretised using the Gauss cubic scheme, while the Laplacian term and pressure term in the momentum equations are discretised using the Gauss linear scheme. And the Euler implicit scheme is adopted for the temporal discretisation.
3.2.3 Boundary conditions

A rectangular computational domain is used in this study, as defined in Figure 3-1 (a). The inflow boundary is 16\(D\) upstream of cylinders 1 and 2. The outflow boundary is 29\(D\) downstream of cylinders 3 and 4 and two side boundaries are 10\(D\) away from the cylinders. The size of the computational domain is selected based on a previous study by Barkley and Henderson (1996) and a domain size dependence study which is detailed in the next section. The initial values for flow velocity and pressure in the whole domain are set to zero and boundary conditions for the governing equations are: (i) at the inlet, a uniform velocity \(U\) is given in \(x\)-direction and the Neumann condition is applied for pressure; (ii) at the outlet, the pressure is set to be zero and the velocity gradients in the streamwise direction are zero; (iii) symmetry boundary conditions are applied at the two lateral boundaries; (iv) no-slip boundary condition is adopted on the cylinder surfaces.

![Figure 3-1 Schematic representation of the four circular cylinder system. The inlet flow \(U\) is from left to right. The cylinders are represented by four circles and labelled 1~4 in clock-wise direction.](image)

3.2.4 Mesh dependency study and model validation

Validations of the present numerical model and a mesh dependence study are carried out for flow past a single cylinder and flow past two cylinders in a tandem arrangement. The effect of mesh size on the accuracy of solution are investigated by simulating the flow past a single cylinder with four different meshes at \(Re=200\) and 300. The four meshes differ from each other in the near wall mesh density as listed in Table 3-2. The sizes of the first layer from the cylinder surface in the radial direction of the cylinder for the four meshes are 0.033\(D\), 0.01\(D\), 0.005\(D\) and 0.001\(D\), respectively. The corresponding non-dimensional distances of the first nodal point to the cylinder surface, \(y^+ = u_f \Delta / \nu\), where \(\Delta\) is the distance from the cylinder surface and \(u_f\) is the friction velocity, are also included in Table 3-2. The computational domain size is 20\(D\times45D\)
with $L_i = 16D$, $L_o = 29D$ and $L_s = 10D$. The distance $L_i$, $L_o$ and $L_s$ are defined in Figure 3-1 (a). Figure 3-1 (b) shows a typical mesh distribution around one cylinder.

Each computation is run for at least $t^* = 650$, where $t^* = Ut/D$, to ensure the establishment of a stable periodic vortex shedding which is quantified by the change in the Strouhal number. Simulation results of force coefficients from a typical case of $Re=200$ are shown in Figure 3-2. It was found that the difference between the Strouhal numbers based on time history of lift coefficient in the periods of 350–650 and 480–650 is less than 0.2%, which is considered to be small enough for the purpose of this study. The Strouhal number was obtained by performing Fast Fourier Transition (FFT) analysis on the lift coefficient, $St = f_s D/U$, where $f_s$ is the frequency of the fluctuating lift force.

Table 3-2 Mesh details and the influence of mesh size on simulation results for a single cylinder at $Re=200$ and $Re=300$

<table>
<thead>
<tr>
<th>Re</th>
<th>Mesh</th>
<th>$\Delta y$</th>
<th>$y^*$</th>
<th>$\bar{C}_D$</th>
<th>$C_L$</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re=200$</td>
<td>Mesh 1</td>
<td>0.033D</td>
<td>1.0</td>
<td>1.399</td>
<td>0.509</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 2</td>
<td>0.010D</td>
<td>0.3</td>
<td>1.360</td>
<td>0.483</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 3</td>
<td>0.005D</td>
<td>0.145</td>
<td>1.357</td>
<td>0.480</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 4</td>
<td>0.001D</td>
<td>0.03</td>
<td>1.355</td>
<td>0.476</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 1</td>
<td>0.033D</td>
<td>1.3</td>
<td>1.457</td>
<td>0.685</td>
<td>0.210</td>
</tr>
<tr>
<td></td>
<td>Mesh 2</td>
<td>0.010D</td>
<td>0.41</td>
<td>1.392</td>
<td>0.642</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td>Mesh 3</td>
<td>0.005D</td>
<td>0.196</td>
<td>1.386</td>
<td>0.638</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td>Mesh 4</td>
<td>0.001D</td>
<td>0.04</td>
<td>1.384</td>
<td>0.633</td>
<td>0.212</td>
</tr>
<tr>
<td>$Re=300$</td>
<td>Mesh 1</td>
<td>0.033D</td>
<td>1.0</td>
<td>1.399</td>
<td>0.509</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 2</td>
<td>0.010D</td>
<td>0.3</td>
<td>1.360</td>
<td>0.483</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 3</td>
<td>0.005D</td>
<td>0.145</td>
<td>1.357</td>
<td>0.480</td>
<td>0.197</td>
</tr>
<tr>
<td></td>
<td>Mesh 4</td>
<td>0.001D</td>
<td>0.03</td>
<td>1.355</td>
<td>0.476</td>
<td>0.197</td>
</tr>
</tbody>
</table>

Table 3-2 lists the variation of the mean drag coefficient, $\bar{C}_D$, the root-mean-square (RMS) lift coefficient $C_L$ and the Strouhal number $St$ with the change of the non-dimensional distance $y^+$. The force coefficients are defined as $C_D = F_x / (\rho U^2 D / 2)$, $C_L = F_y / (\rho U^2 D / 2)$, where $F_x$ is the drag force in the $x$-direction and $F_y$ is the lift force in the $y$-direction. $F_x$ and $F_y$ are obtained by integrating the pressure and shear stress along the cylinder surface. It can be seen that when $y^+$ is less than 0.5, further refinement of the mesh gives little changes (less than 2%) to the force coefficients and to the Strouhal...
numbers. To resolve potential fine flow structures between gaps of the cylinders and accurately predict the forces on downstream cylinders, which was discovered by Papaioannou et al. (2006), Mesh 4, the finest mesh examined is employed in this study, although the results have converged at the less fine Mesh 3.

The effects of computational domain sizes and computational time steps on numerical results are further investigated for flow past a single cylinder at $Re=200$ and the results are listed in Table 3-3. The non-dimensional time step $U \Delta t/D$ of 0.004 and 0.002 based on Mesh 4, where $\Delta t$ is the time step, are used in the time step dependency study. It can be seen from Table 3-3 that the computational results obtained using the two time steps are almost identical. Based on these results, the time step of 0.004 which corresponding to a maximum local Courant number below 0.3 is chosen in this study. An additional simulation (Mesh 5) is carried out by simulating the same flow with a larger domain size but identical minimum mesh size to examine the adequacy of the domain size used in Mesh 4. Mesh 5 has a domain size of twice of that of Mesh 4. Once again, the results obtained using the two domain sizes are almost identical. The force coefficients, which are more sensitive to the change in $\Delta t$ and domain size according to Farrant et al. (2000), also converge at Mesh 4. The numerical results at $Re=200$ are also compared with the data from Han et al.(2013) and Farrant et al.(2000) in Table 3-3 and the agreement is quite good. Since the domain size used in the present study is sufficiently larger than those used by Farrant et al.(2000) and Lam et al.(2010) for four-cylinder array simulations, Mesh 4 is considered to be adequate for this study.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Domain size $(L/L_o/L_s)$</th>
<th>$U \Delta t/D$</th>
<th>$\overline{C_D}$</th>
<th>$C_L$</th>
<th>$C_L$ (peak-to-peak)</th>
<th>$St$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 4</td>
<td>16/29/10</td>
<td>0.004</td>
<td>1.355</td>
<td>0.476</td>
<td>1.35</td>
<td>0.197</td>
</tr>
<tr>
<td>Mesh 4</td>
<td>16/29/10</td>
<td>0.002</td>
<td>1.357</td>
<td>0.477</td>
<td>1.35</td>
<td>0.198</td>
</tr>
<tr>
<td>Mesh 5</td>
<td>32/58/20</td>
<td>0.004</td>
<td>1.330</td>
<td>0.462</td>
<td>1.31</td>
<td>0.197</td>
</tr>
<tr>
<td>Han et al.(2013)</td>
<td>20/30/20</td>
<td>0.005</td>
<td>1.346</td>
<td>-</td>
<td>1.38</td>
<td>0.195</td>
</tr>
<tr>
<td>Farrant et al.(2000)</td>
<td>16/14/10</td>
<td>0.1</td>
<td>1.36</td>
<td>-</td>
<td>1.42</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Table 3-3 Influences of the computational domain size and time step size for the simulation of a single cylinder at $Re=200$, reference data are included for comparison.

The variation of the Strouhal number with the Reynolds number for a single cylinder is compared with available data in literatures in Figure 3-3. The present numerical results compare well with existing experimental and numerical results, especially the curve fitting experimental data of Williamson and Brown (1998), 2D numerical results of Barkley and Henderson (1996) and 3D numerical results of Zhao et al. (2013). The trend line by Williamson and Brown (1998) in Figure 3-3 follows $St = (0.2731+1.1129/
\[ \sqrt{Re} + 0.4821/Re \].

Figure 3-3 Comparison of \( St \) as a function of \( Re \) between the present study and published results.

In addition, the simulation of two tandem cylinders is carried out to further validate the numerical model for flow around multiple cylinders. The computational domain size is \( 20D \times (45+P^*)D \), and the computational mesh density is the same as that of Mesh 4 listed in Table 3-2 and Table 3-3. The Reynolds number is kept at \( Re=200 \) (based on a single cylinder) and five spacing ratios of 1.7, 2, 3, 4 and 6 are considered.

Figure 3-4 compares the present results of Strouhal number (\( St \)) and mean drag coefficient \( \bar{C}_D \) with the published results. The Strouhal number of both cylinders is found to be the same, which is in consistent with the calculations by Meneghini et al. (2001). It is found that the \( St \) at \( P^*=2 \) and 3 is much smaller than at other spacing ratios. The \( \bar{C}_D \) of the downstream cylinder is negative when the gap ratio \( P^* \) is less than 3 because no vortex shedding is observed between the gap and the downstream cylinder is immersed in the wake of the upstream cylinder. The present results are in very good agreement with those by Meneghini et al. (2001), except at \( P^*=4 \), where the present simulation yields slightly larger \( St \) and force coefficients. The difference occurs because \( P^*=4 \) is the critical gap ratio, above which vortex shedding occurs in the gap between the cylinders, and flow is sensitive to the gap ratio at around critical distance (Borazjani and Sotiropoulos, 2009).
For 2D numerical studies of flow past multiple cylinders, the Reynolds number is usually kept to be no more than 200 (Farrant et al., 2000, Han et al., 2013, Lam et al., 2008). The range of Reynolds number in the present study is from 50 to 300. It must be admitted that the three-dimensionality of the flow appears well before $Re=300$ and 3D flow features cannot be captured by 2D numerical models. To address this concern, the two dimensional numerical results are compared with three dimensional results by Tong et al. (2013) for the same flow configuration. The $C_D$ and root-mean-square lift coefficient $C_L'$ from the present 2D simulation of four circular cylinders at $P^*=2$ are compared with the results of 3D simulations by Tong et al. (2013) in Figure 3-5. Generally, the 2D simulation over predicts the RMS lift coefficients, because it assumes the flow to be two-dimensional. The over prediction of the force coefficients by 2D models is also reported by Lei et al. (2000) and Zhao et al. (2013). However, the difference in force coefficients in Figure 3-5 is very small, especially for drag coefficient. Most importantly, the 2D simulations successfully capture the sudden
increase of $C'_L$ at $Re=220$–240, the critical $Re$ for wake transition from Regime 1 to Regime 2, as defined in Tong et al. (2013) in 3D simulations.

The vortex shedding patterns based on 2D and 3D simulations are also compared with each other in Figure 3-6, where the vortex shedding flow is visualized by the contours of the axial vorticity component, $\omega_z$, at $Re=220$ and $Re=240$, where the wake transmits from one regime to another as observed in the three dimensional flow around four-cylinder array (Tong et al., 2013), and at $Re=300$, the highest Reynolds number considered in the present 2D simulations. Both 2D and 3D numerical model results show that the vortex shedding at $Re=220$ is in the in-phase mode, changes to the anti-phase mode at $Re=240$, and keeps to be anti-phase until $Re=300$. The numerical results shown in Figure 3-6 demonstrate that vortex shedding structures revealed by the 2D simulations are very similar to the 3D results for Reynolds numbers up to $Re=300$, although differences in fine details of vorticity contours exist. In addition to the comparisons between 2D and 3D flow features shown in Figure 3-6, the key flow features identified using 2D simulations (presented later on) are also confirmed by 3D simulations at a few selected pairs of Reynolds number and spacing ratio. Due to the large amount of simulations required to map out the flow regimes in the Reynolds
number – spacing ratio plane, the numerical results presented hereafter are obtained from 2D simulations.

![Image: Comparison of instant $\omega_z$ distribution between two-dimensional (Left) and three-dimensional (Right) simulations for four cylinders in a square configuration for $P^* = 2$. Contours vary from $\omega_z = -1$ (cold blue colors & dashed lines) to $\omega_z = 1.0$ (warm red colors & dashed lines).](image)

Figure 3-6

3.3 Results

3.3.1 Wake flow patterns

When four cylinders are placed in a square configuration, the proximity interferences between transverse pairs and the wake interferences between longitudinal pairs bring more complexity to the flow than two cylinders as a result of the interaction among free shear layers from the four cylinders and interactions among the Kármán vortex streets. Simulations are carried out for 12 spacing ratios ($P^*$) of 1.7, 2.0, 2.3, 2.5, 2.7, 3.0, 3.3, 3.5, 3.7, 4.0, 4.3 and 4.5, and Reynolds numbers ranging from 50 to 300 with an interval of 10. The computational domain size is $(20+P^*)D \times (45+P^*)D$. Based on the distinct features of wake structure and force coefficients, seven vortex shedding patterns are identified, which include pattern A of steady wake; pattern B of single bluff-body vortex shedding; pattern C of binary vortex shedding; pattern D of anti-phase vortex shedding; pattern E of in-phase vortex shedding; pattern F of biased vortex shedding.
and pattern G of vortex co-shedding & anti-phase synchronization. It is found that the vortex shedding patterns behind the cylinders are dependent on Reynolds numbers and the spacing ratio $P^*$. Based on extensive simulations carried out in this study, the flow patterns are mapped on the $Re - P^*$ plane as shown in Figure 3-7.

Broadly speaking, vortices are not shed from upstream cylinders and thus the flow in the longitude gaps (the gaps between the upstream and downstream cylinders) resembles the “reattachment” feature in pattern A to pattern D. A similar phenomenon was found at small gap ratios for two cylinders in a tandem arrangement by Igarashi (1981, Igarashi, 1984) and Xu and Zhou (2004). The vortex starts to be shed from the two upstream cylinders in the tandem gaps in pattern E. In pattern F and G, the wake is featured by strong interactions among four Kármán vortex streets.

Figure 3-8 shows instant vorticity contours at $Ut/D = 650$ for these patterns. Only 6 of the 12 calculated spacing ratios are presented in Figure 3-8, and for each of these 6 spacing ratios the vorticity contours at Reynolds numbers with an interval of 20 are given. Different flow patterns will be discussed separately in the following section and additional illustrations of the flow features will also be shown.
Figure 3-8 (a) Instantaneous vorticity contours illustrating the different shedding regimes observed in the flow around four circular cylinders in an in-line square arrangement. Contours vary from $\omega_z = -1$ (cold blue colors & dashed lines) to $\omega_z = 1.0$ (warm red colors & dashed lines).
Figure 3-8 (b) Instantaneous vorticity contours illustrating the different shedding regimes observed in the flow around four circular cylinders in an in-line square arrangement. Contours vary from $\omega_z = -1$ (cold blue colors & dashed lines) to $\omega_z = 1.0$ (warm red colors & dashed lines).
3.3.2 Pattern A: steady wake

The wake flow in pattern A is characterised by four shear layers without vortex shedding, one on either side of each cylinder row as shown in Figure 3-8 at \((Re, P^*) = (60, 2.0 \sim 3.0)\). The shear layers on the inner side of each cylinder row are shorter and thinner than the shear layers on the outer side of the cylinder row due to the interference of flow around two cylinder rows. The lengths of the inner shear bands grow as the spacing ratio increases for the same \(Re\) (See Figure 3-8 at \(Re = 60\) and \(P^* = 2.0\) to \(3.0\)). Pattern A occurs at low Reynolds numbers in the region indicated in Figure 3-7 and the Reynolds number range at a low spacing ratio is found to be wider than that at a high spacing ratio. For instance, at \(P^* = 1.7\), pattern A is in the range from \(Re = 50\) to \(80\), while at \(P^* = 4.5\), pattern A is only found at \(Re = 50\). This indicates that the critical Reynolds number for onset of the primary Kármán vortex increases as the spacing ratio decreases.

![Figure 3-9 Additional illustration of flow pattern B at \((Re, P^*) = (80, 1.7)\) along with the force coefficients on downstream cylinders Left, Instantaneous vorticity contours, which vary from \(\omega_z = -1\) (cold blue colors & dashed lines) to \(\omega_z = 1.0\) (warm red colors & dashed lines). Right, time histories force coefficients; dashed thin line, \(C_{D3}\); dashed thick line, \(C_{D4}\); solid thin line, \(C_{L3}\); solid thick line, \(-C_{L4}\).](image)

3.3.3 Pattern B: single bluff-body vortex shedding

Pattern B is characterised by vortex shedding from the opposite sides of the four-cylinder system and the four cylinders behave as a single bluff-body with only one street of Kármán vortex. A flow visualization example for pattern B is shown in Figure 3-8 at \((Re, P^*) = (80, 2.0)\). Figure 3-9 shows the vortex shedding and time series of the force coefficients on the two downstream cylinders for pattern B at \((Re, P^*) = (80, 1.7)\). It can be seen that the wake of the four-cylinder bluff body is primarily dominated by the vortices shed from outer shear layers. The location of the strong vortex interaction is quite far downstream the four-cylinder system. The interactions between two inner shear layers and between the inner and outer shear layers of each cylinder row are really weak and the weak vortices shed from the weak interactions are dissipated quickly. This is due to the “near-wall effect”, as observed by Xu et al. (2003) and Sumner (2010) for
two cylinders in the side-by-side configuration, i.e. the transverse gaps between cylinders are too small to allow vortex shedding to occur.

Pattern B occurs for a narrow band of Reynolds numbers for spacing ratios less than 3 as shown in Figure 3-7. When the spacing ratio exceeds 3, the shear layers from the inner sides of the cylinders are strong enough to form vortices by interacting with the corresponding outer shear layers, resulting in the formation of Kármán streets behind downstream cylinders. The lower bound spacing ratio of 1.7 for pattern B is found to be larger than that of a two side-by-side cylinder system which is 1~1.2 (Sumner, 2010). This is partly because the flow through the gap between two cylinder rows in the four-cylinder system is weaker than that through the gap between the two side-by-side cylinders.

The drag and lift coefficients on the k-th cylinder are defined as $C_{Dk}$ and $C_{Lk}$, respectively in the discussion. Time histories of force coefficients on the two rear cylinders are shown in Figure 3-9. In Figure 3-9, the negative value of the lift coefficient on cylinder 4, i.e. $-C_{L4}$ is plotted in order to compare the magnitudes of $C_{L3}$ and $C_{L4}$. It can be seen that both the drag and the opposite lift coefficients of cylinder 4 are in anti-phase with their counterparts of cylinder 3, due to the fact that the vortices are shed from cylinders 3 and 4 alternatively. The repulsive lift force between the two downstream cylinders are similar to that observed for two cylinders in a side-by-side configuration (Farrant et al., 2000), as the lift coefficients of the two downstream cylinders are in opposite directions.

3.3.4 Pattern C: binary vortex shedding

A single vortex street is still observed in the combined wake of the four cylinders in pattern C, binary vortex shedding, in the region indicated in Figure 3-7. Pattern C occurs at relatively small spacing ratios over a wide range of Reynolds numbers. In pattern C, vortices are shed from the inner sides of the two downstream cylinders, and only a single vortex street is found in the wake of the cylinders because the two vortices of the same sign merge to form one vortex soon after they are shed from the cylinders in one vortex shedding period. The vortices that are shed from the inner side of one of the rear cylinders travel across the centerline of the system (i.e. $y=0$ line) and merge with those vortices of the same sign shed from the outer side of the other cylinder, such as cases of $(Re, P^*) = (160~220, 2.0)$ in Figure 3-8.
(a) regular binary vortex shedding

Figure 3-10 Instantaneous vorticity contours in a vortex shedding period illustrating the regular and irregular mode of binary vortex shedding for patterns C at \((Re, P^*) = (110, 2.5)\) and \((110, 2.0)\). Contours vary from \(\omega_z = -1\) (cold blue colors & dashed lines) to \(\omega_z = 1.0\) (warm red colors & dashed lines).

Pattern C may eventually evolve into a regular mode with a single binary vortex
street (for cases of $P^* = 2.3$ at $Re=140$, $P^* = 2.5$ at $Re=100$–$110$, and $P^* = 2.7$ at $Re=90$). The binary vortex street is characterised by a large-scale combined wake comprising of vortices of opposite signs in a staggered arrangement. Each wake vortex is the combination of two vortices of the same sign shed from upper and downside cylinders, respectively. This in-phase synchronization of the vortex street was firstly reported by Williamson (1985) at $Re=200$, $P^* = 3.0$–$5.0$ for two cylinders in a side-by-side configuration, and named binary vortex shedding to illustrate the two rotating like-signed vortices. The formation of the binary vortex shedding is also presented in Figure 3-10 (a) in a vortex shedding period at $(Re, P^*) = (110, 2.5)$. Different from the vortices in a Kármán street, each of the binary vortices in Figure 3-10 (a) from $x/D = 7$ to $17$ is the combination of two vortices. For example, two vortices from the top sides of the two downstream cylinders are merging from $Ut/D = 552$ to $553$ to form a new vortex, while the two vortices from the bottom sides of the two downstream cylinders are merging from $Ut/D = 556$ to $557$ in Figure 3-10 (a). The binary vortices are convected downstream forming two rows of vortices, which are staggered at the two sides of the centerline of the system.

For most of the cases in pattern C, the vortex flow is very irregular. The irregular mode of binary vortex shedding flows of pattern C in a typical vortex shedding period is shown in Figure 3-10 (b) at $(Re, P^*) = (110, 2.0)$. It is seen that the vortices of the same sign shed from two rear cylinders also merge to form one vortex, but the vortex street is rather chaotic with many small vortices. The vortex shedding from two cylinder rows is generally in the in-phase mode, similar to that in the regular vortex street in pattern C, but the regularity quickly disappears as the vortices are convected downstream and many smaller vortices are generated in the wake region from about $x/D = 5$ to $15$, as also seen in Figure 3-8 at $(Re, P^*) = (160$–$220, 2.0)$. These small scale vortices are eventually merged into large scale ones or simply disappears beyond $x/D = 20$.

Figure 3-11 compares the time histories of force coefficients on the two downstream cylinders between the regular and irregular modes of binary vortex shedding shown in Figure 3-10. The negative value of the lift coefficient on the right bottom cylinder, $-C_{L4}$ is presented in Figure 3-11 in order to compare the lift coefficients of cylinders 3 and 4 conveniently. It is observed that the amplitudes of both drag and lift coefficients in the irregular mode are greater than their counterparts in the regular mode. The regular binary vortex shedding for $(Re, P^*) = (110, 2.5)$ is sustained for a non-dimensional time
of more than 200, indicating very stable vortex shedding. This is different from the experimental observations for two side-by-side cylinders by Williamson (1985), where the binary vortex shedding was found to be in an unstable and intermittent state and changed back to the dominated anti-phase vortex shedding frequently. Pattern C disappears as spacing ratio exceeds 3.7 and is essentially a transition pattern from a single Kármán street to the two parallel vortex streets in Pattern D.

Pattern D: anti-phase vortex shedding

Pattern D is characterised by two parallel vortex streets synchronized in an anti-phase fashion. In pattern D, vortices are shed simultaneously from the two downstream cylinders at the same frequency. Typical pattern D flows can be seen in Figure 3-8 for \((Re, P^*) = (240~300, 2.0), (Re, P^*) = (120~160, 2.5)\) and \((Re, P^*) = (80~120, 3.0)\), to just name a few. In pattern D, vortex shedding does not occur behind the two upstream cylinders. The combination of the vortices that occurs in pattern C does not occur in pattern D, resulting in two distinct Kármán vortex streets in the wake of the four cylinders symmetrically distributed at the two sides of the centerline. Pattern D is observed for the spacing ratio range considered in this study as shown in Figure 3-7. The anti-phase vortex shedding in the case of two cylinders in a side-by-side arrangement was studied in details (Meneghini et al., 2001, Sumner, 2010) and was identified as one of the main flow patterns behind a pair of bluff bodies (Williamson, 1985). Peschard and Le Gal (1996) also named this flow feature as locking and phase opposition regime. Due to its unique feature and prevalence, the anti-phase vortex shedding was among the first studied flow features for four cylinders (Farrant et al., 2000). Typical time histories of lift force coefficient on downstream cylinders are given in Figure 3-12 for \((Re, P^*) = (300, 2.0)\) and \((Re, P^*) = (120, 3.0)\), along with the wake flow structures in a vortex shedding period for the latter case. The wake flow feature in
pattern D keeps stable, as illustrated in Figure 3-12; because the wake configuration keeps its symmetric pattern for long computational time period (more than 500) and for long distance in the wake (more than 29D). It is also observed that the drag forces on cylinders 3 and 4 are perfectly the same, and the negative lift coefficient on cylinder 4 \(-C_{L4}\) is also the same as \(C_{L3}\), resulting in a zero total lift on cylinders 3 and 4. This behavior of the force coefficient is consistent with the observed flow features.

Figure 3-12 The force coefficients on downstream cylinders (Above) and the snapshots of instantaneous vorticity contours in a vortex shedding period (Bottom), when pattern D takes place for flow around four circular cylinders in an in-line square arrangement; dashed thin line, \(C_{D3}\); dashed thick line, \(C_{D4}\); solid thin line, \(C_{L3}\); solid thick line, \(\text{\(-}\ C_{L4}\); lines of cylinder 3 are hidden by those on cylinder 4. Contours vary from \(\omega_z = -1\) (cold blue colors & dashed lines) to \(\omega_z = 1.0\) (warm red colors & dashed lines).

3.3.6 Pattern E: in-phase vortex shedding and in-phase escape

The Re–\(P^*\) region where pattern E occurs is indicated in Figure 3-7. Pattern E is characterised by in-phase vortex shedding. In Figure 3-8 wake flows for \((Re, P^*) = (140\sim200, 3.0)\) and \((Re, P^*) = (100, 3.5)\) are examples of pattern E. Close to the
cylinders, the vortex shedding flows from the two downstream cylinders are generally in an in-phase mode and are anti-symmetrically distributed with the centerline. However, the vortices later evolve into a staggered distribution in the wake starting from about \(6D\) away from the downstream cylinders. Different from the anti-phase vortex shedding in pattern D, which is featured with two stable trains of Kármán vortices in the combined wake, there are two types of harmonic vortex synchronizations for the in-phase vortex shedding in Pattern E, with one and three streets of Kármán vortices, respectively. Besides, the wake flows in pattern E at low spacing ratios \((2.3 \leq P^* \leq 2.7)\) are in an irregular mode (See Figure 3-8 for \(P^*=2.5\) at \(Re=220\sim300\)). Based on the wake flow structure, the in-phase vortex shedding pattern E is further divided into two sub-patterns E1, where the vortex shedding does not occur from the upstream cylinder and E2 where vortex shedding occurs from the two upstream cylinders. It should be noted that the onset of vortex shedding from two upstream cylinders is identified by the combination of variations of RMS lift coefficient on downstream cylinders with Reynolds number and flow visualization of wake flow features behind two upstream cylinders. It will be shown later on that the variation of RMS lift coefficient on downstream cylinders experiences a sharp increase with Reynolds number when onset of vortex shedding from upstream cylinders occurs. The sharp increase in RMS lift coefficient on downstream cylinders is induced by the interaction between downstream cylinders and vortices shed from upstream cylinders.

One harmonic mode for Pattern E1 is characterised by a single vortex street, which is found for \((Re, P^*) = (200, 2.5), (100, 3.5), \) and \((80, 4.0),\) in Figure 3-8. Only one vortex street is formed because two vortices of the same sign shed from the two downstream cylinders combine to form one vortex in one vortex shedding period. This flow feature is similar to the binary vortex shedding in Pattern C, but with much stronger vortex shedding from the inner sides of the two downstream cylinders. While the wake in Pattern E1 is found to be stable at \((Re, P^*) = (100, 3.5)\) and \((Re, P^*) = (80, 4.0),\) the vortex street for \((Re, P^*) = (200, 2.5)\) changes intermittently between anti-phase vortex shedding pattern D and in-phase vortex shedding pattern E1, as shown in Figure 3-13. It is seen in the time history of the force coefficients in Figure 3-13 that the lift coefficients on the downstream cylinders 3 and 4 are in anti-phase with each other before instant \(\Box\) \((C_{L3} \text{ and } -C_{L4} \text{ are the same}),\) and change to be in-phase at instant \(\Box\) \((C_{L3} \text{ and } -C_{L4} \text{ are staggered}).\) The change in forces corresponds to the change in the
flow patterns as shown in the vorticity contours in Figure 3-13. The intermittent switch between anti- and in-phase vortex shedding is consistent with the observation reported by Williamson (1985) for two cylinders in a side-by-side configuration. The strong modulations of time histories of force coefficient on downstream cylinders observed in Figure 3-13 are induced by the interactions between the downstream cylinder and the wakes behind upstream cylinders. The wakes behind the upstream cylinders are found to be highly unstable but no regular vortex shedding from the upstream cylinders are observed at this spacing ratio and Reynolds number \((Re, P^*) = (200, 2.5)\).

![Figure 3-13](image)

Figure 3-13 The switchover from anti-phase vortex shedding to in-phase vortex shedding of Pattern E1 for flow around four circular cylinders in an in-line square arrangement. Above, time histories force coefficients; dashed thin line, \(C_{D3}\); dashed thick line, \(C_{D4}\); solid thin line, \(C_{L3}\); solid thick line, \(-C_{L4}\). Bottom, Instantaneous vorticity contours in a vortex shedding period, which vary from \(\omega_z = -1\) (cold blue colors & dashed lines) to \(\omega_z = 1.0\) (warm red colors & dashed lines).

The harmonic mode for flow pattern E2 is characterised by triple streets of vortices in the wake such as those for \((Re, P^*) = (140–200, 3.0)\) in Figure 3-8 and \((Re, P^*) =
Figure 3-14 Additional illustration of flow Pattern E (three streets vortex synchronization) at \( Re = 280 \) and \( P^* = 2.7 \) along with the force coefficients on downstream cylinders. Above, time histories force coefficients; dashed thin line, \( C_D^3 \); dashed thick line, \( C_D^4 \); solid thin line, \( C_L^3 \); solid thick line, \( -C_L^4 \).

Below, Instantaneous vorticity contours in a vortex shedding period, which vary from \( \omega_z = -1 \) (cold blue colors & dashed lines) to \( \omega_z = 1.0 \) (warm red colors & dashed lines).

(280–300, 2.7), and Figure 3-14 shows a detailed flow field of pattern E with three streets vortex synchronization in a vortex shedding period at \((Re, P^*)=(280, 2.7)\) along with the force coefficients on downstream cylinders. Two streets of vortices are staggered distributed at both sides of the cylinder array, and one street of vortices is distributed along the centerline. The vortices at the centerline are mainly generated from the inner sides the four cylinders and each one evolves into a binary vortex including two like-signed vortices at about \( x/D = 10\text{–}18 \). The vortices in the central street are generally weaker than those in the side streets. The maximum absolute lift forces are staggered distributed, similar to those observed in Pattern B and Pattern C. Obvious modulation of lift coefficients on both cylinders is observed, suggesting the existence of contributions from a different flow mechanism from the dominant vortex shedding process from the rear cylinders. This is likely caused by the vortex shedding from upstream cylinders. It also observed that every second peak in the time histories of drag coefficient is subdued, again likely due to the vortex shedding from upstream cylinders.
To our best knowledge, this flow feature of triple vortex shedding streets has not been reported in previous studies.

Another interesting flow feature observed in pattern E is the vortex shedding from upstream cylinders. It is observed that vortex shedding from the two upstream cylinders occurs in phase (except the case of $P^* = 4.5D$ as the flow pattern E is not found for the largest spacing ratio) and vortices shed from two upstream cylinders pass over downstream cylinders from either the top or the bottom side of the downstream cylinders (in phase). The boundary between E1 (no-vortex shedding from the two upstream cylinders) and E2 (vortex shedding from the two upstream cylinders) is identified as the critical $Re - P^*$ for vortex shedding from upstream cylinders in Figure 3-7. The instant flow fields, when the onset of vortex shedding from upstream cylinders takes place for a few selected spacing ratios, are shown in Figure 3-15. Detailed flow features in one typical vortex shedding period at $(Re, P^*) = (210, 2.5)$ and $(Re, P^*) = (130, 3.0)$ are given in Figure 3-16. The directions of movement of vortices shed from two upstream cylinders are indicated by two arrows in Figure 3-15 and Figure 3-16, where the in-phase vortex shedding from two upstream cylinders are clearly observed. The vortices shed from the upstream cylinders escape from the gaps between up- and down-stream cylinders and pass over corresponding downstream cylinders from the same side of the downstream cylinders, either from the top or bottom sides of cylinder 3 and cylinder 4 simultaneously. The instants corresponding to vortex escapes from longitudinal gaps between upstream and downstream cylinders are identified as $Ut/D = 530$ for $(Re, P^*) = (210, 2.5)$ in Figure 3-16 (a) and at $Ut/D = 554$ for $(Re, P^*) = (130, 3.0)$ in Figure 3-16 (b). The “in-phase” vortex shedding and “escape” of the gap flow are referred to as the “in-phase escape” regime to represent the two main flow features in pattern E. The “in-phase escape” flow feature is identified in the present study and has not been reported previously. At large $P^* (P^* \geq 4.5)$, the in-phase escape feature disappears because of the weakening of proximity interferences.

### 3.3.7 Pattern F: biased vortex shedding

The biased vortex shedding Pattern F is found at higher Reynolds numbers than pattern E and at intermediate spacing ratios ($2.8 \leq P^* \leq 4.3$) as shown in Figure 3-7. This flow pattern is characterised by a gap flow slightly biased towards one side of longitudinal centerline, which occurs in both anti-phase vortex shedding dominated wake (see $P^* = 3.0$ at $Re = 280$, and $P^* = 3.5$ at $Re = 240\sim280$ in Figure 3-8) and in-phase
vortex shedding dominated wake (see $P^* = 3.0$ at $Re = 220 \sim 260$ and $P^* = 3.5$ at $Re = 120$ in Figure 3-8). It can also be observed that vortices are shed from all of the four cylinders in Pattern F. A sketch of vortex formation, vortex shedding and interaction among vortices in the wake for pattern F is shown in Figure 3-17, where two Kármán vortex streets are observed, with one in the wake of the bottom pair of cylinders being wider than that from the top pair. The vortices that are shed from top side of the bottom downstream cylinder travel cross the longitude centerline and dissipate gradually. The biased flow pattern was also reported as one of the main flow features exhibited by two cylinders in a side-by-side arrangement in previous investigations (Zhou et al., 2002, Xu et al., 2003, Williamson, 1985, Sumner, 2010).

In this study, the biased vortex shedding in Pattern F is found to stay biased towards one side at relatively low Reynolds number range during the simulation period (for instance cases of $P^* = 3.0$ at $Re = 210 \sim 260$ and $P^* = 3.5$ at $Re = 210 \sim 220$), and switches from one side to another side during the simulation period at relatively large Reynolds numbers for the same spacing ratio (for instance cases of $P^* = 3.0$ at $Re = 270 \sim 300$ and $P^* = 3.5$ at $Re = 230 \sim 300$). Zhou et al. (2002) and Sumner (2010) found that, for two side-by-side cylinders, the biased flow switched from one side to another side after tens or hundreds (or a few orders) of vortex shedding periods. There are also experimental evidences that the biased vortex street never changes its deflection direction for flow around two side-by-side cylinders. To confirm this, simulation period is extended to
$t^* = Ut/D = 1950$, triple of the original period of $Ut/D = 650$ for those cases of $(Re, P*) = (210-260, 3.0)$. The simulation results (not shown here due to page limit) suggest that the biased vortex remains in one deflection direction up to $Ut/D = 1950$ for cases of $(Re, P*) = (220-260, 3.0)$ while three streets wake (pattern E2) develops for case of $(Re, P*) = (210, 3.0)$ at about $t^*=800$. This is because case $(Re, P*) = (210, 3.0)$ is on the boundary between pattern E and pattern F where the wake is bistable.

(a) Pattern E, the in-phase escape at $(Re, P*) = (210, 2.5)$

(b) Pattern E, the in-phase escape at $(Re, P*) = (130, 3.0)$

Figure 3-16 Additional illustration of the “in-phase escape” of vortex shedding from upstream cylinders in Patterns E at $Re = 210$, $P* = 2.5$ (Above) and at $Re = 130$, $P* = 3.0$ (Bottom) in about one vortex shedding period. Instant vorticity contours vary from $\omega_z = -1$ (cold blue colors & dashed lines) to $\omega_z = 1.0$ (warm red colors & dashed lines).

Figure 3-17 Schematic of biased vortex shedding from four circular cylinders in the in-line square configuration.
Figure 3-18 Time histories of the force coefficients on downstream cylinders at \((Re, P^*) = (240, 3.0)\) when switchover does not occur to the biased flow; dashed thin line, \(C_{D3}\); dashed thick line, \(C_{D4}\); solid thin line, \(C_{L3}\); solid thick line, \(-C_{L4}\).

The force coefficients on downstream cylinders at \((Re, P^*) = (240, 3.0)\) where the flow is biased towards cylinder 4 (see Figure 3-8) and the switchover does not occur are shown in Figure 3-18. It is seen that \(C_{D4}\) are obviously larger than \(C_{D3}\) and amplitudes of \(C_{L3}\) are consistently larger than those of \(-C_{L4}\). The force coefficients and flow...
visualizations for the switchover of vortex street bias at \((Re, P^*) = (280, 3.5)\) are given in Figure 3-19, where the flow changes its biased direction at about \(Ut/D = 520\). Before \(Ut/D = 520\), the flow biases towards two top cylinders, and after \(Ut/D = 520\) it biases towards two bottom cylinders. The switchover is found to be more frequent at higher \(Re\) and higher spacing ratios. Similar to two cylinders in a side-by-side configuration, the cylinders towards which the vortex shedding flow biases to have narrower near-wakes than other two cylinders in the system, resulting higher drag coefficients on these two cylinders. No obvious trend is found for lift coefficient on downstream cylinders and it appears that the amplitude of lift coefficient on downstream cylinders is independent of the deflection direction of the wake.

### 3.3.8 Pattern G: Co-shedding & anti-phase synchronization

In pattern G, the wake flow is dominated by vortex co-shedding from both upstream and downstream pairs of cylinders and the vortex streets from the top pair and the bottom pair of cylinders are synchronized in anti-phase mode. The cylinder wake behind the top row of cylinders is symmetrical about longitudinal centerline that separates the top row and bottom row cylinders. Pattern G is observed at large spacing ratios \((P^* \geq 3.5)\) in Figure 3-7. The flow visualizations are shown in Figure 3-8 for \((Re, P^*) = (120–300, 4.0)\) and \((Re, P^*) = (100–300, 4.5)\). The flow pattern G is very similar to pattern D, except that vortices are also shed from the upstream two cylinders in pattern G. Also, the force coefficients on downstream cylinders, illustrated in Figure 3-20, exhibit the same pattern as in that pattern D, except that there are obvious modulations in the drag coefficients on downstream cylinders.

![Figure 3-20](image)

**Figure 3-20** The force coefficients on downstream cylinders at pattern G; dashed thin line, \(C_D^3\); dashed thick line, \(C_D^4\); solid thin line, \(C_L^3\); solid thick line, \(−C_L^4\); lines of cylinder 3 are hidden by those on cylinder 4.

### 3.3.9 The boundaries of these flow patterns

It should be noted that the boundaries among above mentioned flow patterns are not
strictly fixed, because the flow may switch from one pattern to its neighbor pattern intermittently at a Reynolds number or the gap ratio close to or on the boundary. One of such examples is the wake flow at $Re=180$~$200$, $P^*=2.5$ which lies on the boundary between Patterns D and E as shown in Figure 3-7. While the current classification attributes the case of $Re=180$ to pattern D as anti-phase vortex shedding and $Re=200$ to pattern E as in-phase vortex shedding, it is found the in-phase vortex shedding does intermittently appear at $Re=180$ and the anti-phase vortex shedding is also found at $Re=200$ (see Figure 3-13).

Figure 3-21 Comparison of force coefficients on the downstream cylinders at different spacing ratios; Above, RMS lift coefficient; Bottom, mean drag coefficient. The legend denotes the spacing ratios, for instance, s1.7 represents $P^*$=1.7.

3.3.10 Force coefficients

The root-mean-square lift coefficient ($C'_L$) and average drag coefficient ($\overline{C_D}$) on the downstream cylinders as a function of $Re$ for several $P^*$ are shown in Figure 3-21 and compared with those of a single cylinder in steady flow. The force coefficients on
cylinder 3 and cylinder 4 are almost the same in all flow patterns, except in Pattern F where the biased vortex shedding takes place. Here the values shown in Figure 3-21 are the averaged lift and drag coefficients on cylinder 3 and cylinder 4 to eliminate the influences of biased vortex shedding and to facilitate the comparison.

In Figure 3-21, $C_L$ is generally in the trend of increase with the increase of $Re$ and spacing ratio. Sharp increases in RMS lift coefficient are observed at most of the spacing ratios covered in the present study. These sharp increases in RMS lift coefficient are believed to be associated with flow pattern transitions around the cylinder array. The sharp increases in RMS lift coefficient at $(Re, P^*) = (230, 2.0), (120, 2.5)$ and $(100, 2.7)$ are due to wake transitions from the binary vortex shedding pattern $C$ to the anti-phase vortex shedding pattern $D$. The sharp increases in RMS lift coefficient at $(Re, P^*) = (210, 2.5), (180, 2.7), (130, 3.0)$ and $(80, 4.0)$ correspond to the onset of vortex shedding from two upstream cylinders in Pattern E. The vortices shed from upstream cylinders impinge on the rear cylinders, resulting in the increases in RMS lift coefficient on downstream cylinders. It is also observed that the RMS lift coefficients $C_L$ on downstream cylinders is generally smaller in pattern A~C, slightly larger in pattern D, and much larger in pattern E~G than its counterpart of a single cylinder.

For drag coefficients $C_D$ on downstream cylinders, the jump at boundary line between E1 and E2 is also apparent for $P^* =2.5~4.0$, while no such feature is found for $P^* =1.7$ and 2.0 because Pattern E does not appear at these two spacing ratios for the $Re$ range considered. For all spacing ratios, the drags stay below 0.3 and decrease with the increase of $Re$ smoothly at pattern A~D. After the sudden jump of the drag coefficient at the onset of vortex shedding from upstream cylinders, the drag increases continuously at slow rates with the increasing $Re$ in pattern E, but the drag coefficient starts to drop in pattern F (for $P^* = 3.0$ and $P^* = 4.0$ after $Re=120$ and $Re=200$, respectively). In total, the mean drag coefficients on downstream cylinders are much smaller than those of a single cylinder.

The forces on upstream cylinders are not shown in this paper. Generally speaking, forces on the upstream cylinders are less influenced by the cylinder proximity and flow interference than the downstream ones in the four-cylinder system investigated. Although the jumps at pattern E are also observed on upstream cylinders, but the
The magnitude of forces are generally equivalent to the forces on a single cylinder in steady flow as shown in Figure 3-21, especially for drag forces.

![Figure 3-22](image)

**Figure 3-22** Comparison of total force coefficients as a function of spacing ratios on the four cylinders’ system at different Reynolds number; Above, root-mean-square lift coefficient; Bottom, mean drag coefficient. The markers denotes the flow patterns, for instance, D represents pattern D.

In some engineering applications, for instance the forces on the legs of a Tension Leg Platform for oil and gas exploitations, the total force on the system is equally important as forces on a cylinder in the system. Thus, the total force coefficient is given as a function of spacing ratios for Re=100, 200 and 300 in Figure 3-22. The total lift and drag forces $F_{x,T}$ and $F_{y,T}$ are calculated by integrating the pressure and shear stress on the surfaces of all the four cylinders. Then the total drag and lift force coefficients are defined as $C_{D,T} = F_{x,T} / (\rho U^2 D / 2) / 4$, $C_{L,T} = F_{y,T} / (\rho U^2 D / 2) / 4$ to facilitate the comparison with forces on a single cylinder in steady current. In Figure 3-22, the flow patterns are also marked by a corresponding letter. The bundle of four-cylinder system experiences much larger root-mean-square lift forces in patterns E and F (in-phase and biased vortex shedding) than in pattern C, D and G, while in pattern C, D and G, $C'_{L,T}$ are close to zero for all spacing ratios and Reynolds numbers. This is consistent with the flow features.
observed in each pattern. From the mean drag coefficient chart, the instability of forces in pattern E and F is also obvious, with a sudden jump from pattern D to E. In contrast, the forces are more stable in pattern C, D and G for different spacing ratios.

Although the Reynolds number concerned in this study is relatively low, the vortex shedding features observed are expected to exist at high Reynolds numbers. This is because some of the flow features discovered in this study share similarities to flow features around two cylinders observed from independent studies at high Reynolds numbers. For instance, the biased vortex shedding at switchovers between deflected sides was observed at $Re=5.5 \times 10^4$ for two cylinders (Alam et al., 2003) in a side-by-side arrangement. The bluff-body vortex shedding, the biased vortex shedding and the two synchronized vortex streets were observed for two side-by-side cylinders in subcritical flow (Zdravkovich, 1987); Vortex co-shedding from upstream and downstream cylinders were found in turbulent gas flow (Ishigai et al., 1972). Flow around a four-cylinder array at large Reynolds number will be further studied.

3.4 Conclusions

Steady uniform flow around a four-cylinder array in a square arrangement is simulated numerically at relatively small Reynolds numbers (defined using flow velocity $U$ and cylinder diameter $D$) ranging from 50 to 300 with an interval of 10 and spacing ratios (defined $L/D$, with $L$ being the centre-to-centre distance between two adjacent cylinders) from 1.7 to 4.5. Wake characteristics behind cylinders and hydrodynamic forces on the cylinders are investigated by carrying out a series of numerical tests. It is found that vortex shedding around cylinders, wake characteristics and hydrodynamic forces on the cylinders are dependent on Reynolds numbers and spacing ratios. A total of seven flow regimes are identified and are mapped onto the Reynolds number – spacing ratio plane. Each of the seven flow regimes, which are referred to as Pattern A to Pattern G for the convenience of discussion, has distinguished flow features, resulting from the interactions among shear layers around the cylinders, Kármán vortex streets behind downstream cylinders and vortices shed around cylinders. Some of the flow features around the four-cylinder array, such as the single bluff-body vortex shedding pattern B, binary vortex shedding pattern C, in-phase vortex shedding pattern D, biased vortex shedding pattern F, and vortex co-shedding & anti-phase synchronization pattern G, share similarities with flow features exhibited by flow around two side-by-side cylinders and flow around two cylinders of a tandem arrangement.
arrangement. Some of the flow features, such as triple vortex streets and the in-phase escape of vortices at the onset of vortex shedding from upstream cylinders found in the in-phase vortex shedding pattern E, are unique to the four-cylinder system and have not been reported previously. The detailed flow features and force characteristics on the cylinders are summarized below.

No vortex shedding is found in pattern A at small Reynolds numbers (less than 60 for $3.3 \leq P^* \leq 4.5$, 70 for $2.0 \leq P^* \leq 3.0$ and 70 for $P^* = 1.7$). Pattern B and pattern C are featured with a single wake at spacing ratios smaller than 3.3 and Reynolds numbers greater than approximately 70. The difference between pattern B and pattern C is that vortices are found to shed from the inner side shear layers in pattern C. Two parallel streets of Kármán vortex, which are symmetrically distributed along the longitude centerline, dominate the wake of flow pattern D. In the in-phase vortex shedding of pattern E, one (pattern E1) and three (pattern E2) streets of Kármán vortex synchronization are detected. Biased vortex shedding, one of the main flow features for two cylinders in a side-by-side configuration, is also present in flow around the four circular array in pattern F, and the switchover from one deflected side to the other is also observed. In pattern G, located at large $Re$ and large spacing ratios, the wake is featured by vortex co-shedding from both up- and down-stream cylinders, which are symmetrically distributed.

1. Vortex shedding from the upstream cylinders is observed in patterns E, F and G at large spacing ratios when the Reynolds number is greater than about 80. The onset of vortex shedding from upstream cylinders shares similar features with those of the in-phase mode and is referred to as “in-phase escape” (except the largest spacing ratio considered), where the wake flow immediately behind the upstream cylinders moves to the same direction in order to pass over the downstream cylinders when the onset of vortex shedding takes place.

2. Hydrodynamic forces on the downstream cylinders are found to be relatively small in flow pattern A to pattern D where vortex shedding from upstream cylinders does not take place. The hydrodynamic forces on the downstream cylinders jump to large values at pattern E due to vortex shedding from upstream cylinders. The root-mean-square lift force on downstream cylinders increases consistently with the increases of $Re$ and spacing ratio. The drag forces on the rear cylinders appear to decrease slightly at large spacing ratios ($\geq 3.0$) and at large $Re$ ($\geq 200$). The total lift coefficient
on the four-cylinder array as a whole is close to zero at patterns D and G due to the anti-phase pattern of the flow field, while it is much larger at patterns E and F where the in-phase or biased vortex shedding dominates the wake.

Acknowledgments

This work was supported by Australian Research Council Discovery Grant (Project ID: DP110105171) and by iVEC through the use of advanced computing resources located at iVEC@Murdoch. F.T. would like to acknowledge the support of the Australian Government and the University of Western Australia by providing SIRF and UIS scholarships.

3.5 References


Chapter 3

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Chapter 4

The vortex shedding around four circular cylinders in an in-line square configuration

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Abstract: This paper presents a numerical study of three-dimensional (3-D) vortex shedding flow in the wake of four circular cylinders in a square configuration with a constant space-to-diameter ratio of 2. Numerical tests are carried out for Reynolds number (Re) in the range from 100 to 500. Four wake flow regimes are identified at this spacing ratio. Regime 1 (100 ≤ Re ≤ 220) is characterized by the inclination of weak spanwise vortices and weak streamwise vortices, where the wake behind four cylinder array shares similar features to that behind a single cylinder with a large equivalent diameter. It is observed that the onset of three-dimensionality in the wake behind four cylinder array occurs at lower Re than that behind a single cylinder. Regime 2 (240 ≤ Re ≤ 300) is characterized by the appearance of the regular wavy spanwise vortices.

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vortices and rib-shaped streamwise vortices. The wavelength of the spanwise vortices is about 1.2 and the wake flow is similar to the transition mode B of a single cylinder. Regime 3 \((320 \leq \text{Re} \leq 380)\) is characterized by severe vortex dislocations in the wake of the cylinders and regime 4 \((400 \leq \text{Re} \leq 500)\) is characterized by the absence of vortex dislocations and the strong streamwise vortices. The flow between the upstream and the downstream cylinders is predominantly two-dimensional (2-D) in regimes 1, 2 and 3 and becomes 3-D in regime 4. Physical mechanisms responsible for different flow regimes are proposed and discussed in details. Significant changes in the root-mean-square (RMS) force coefficients, wake formation length and phase angle of the lift coefficients on the downstream cylinders are observed when the flow transits from one regime to another.

### 4.1 Introductions

Vortex shedding from a bluff body has been studied extensively in the past decades due to its relevance to engineering applications and its richness in fundamental fluid mechanics. It has been well understood that the flow regimes in the wake of a circular cylinder depend greatly on Reynolds number defined by \(\text{Re} = \frac{U D}{\nu}\), where \(D\) is the diameter the cylinder, \(U\) is the free stream velocity and \(\nu\) is the kinematic viscosity of the fluid. Flow in the wake of a circular cylinder can be categorized into several regimes based on Reynolds numbers (Williamson, 1996a). When Reynolds number is small \((\text{Re} = 47\sim140-190)\), the vortex flow is two-dimensional (2-D). As Reynolds number is increased to around 180\sim194, secondary instability in the wake flow field appears. The three-dimensional instability, which is characterized by the inception of streamwise vortex loops with a spanwise wavelength of 3\~4 times of the cylinder diameter \((D)\) and a sudden decrease in Strouhal frequency, is referred to be mode A (Williamson, 1996b, Williamson, 1988). The spanwise wavelength of the streamwise vortex pairs decrease from around 3\~4\(D\) to approximately 1\(D\) as Reynolds number is further increased to 230\~250, where the mode A instability gradually gives way to the so called mode B instability. The wake fulfils the transition from 2-D to 3-D in mode B (Williamson, 1996a). Comprehensive reviews of flow past a single cylinder can be found in Bearman(1984), Williamson(1996a), Sumer and Fredsøe (1997) and Williamson and Govardhan(2004).

Compared with that of a single cylinder, research concerning flow around multiple cylinders is relatively limited. Zdravkovich (1977, 1987) pointed out that the flow
features around multiple cylinders could be very different from those around a single cylinder. It is generally believed that the vortex shedding pattern, the force coefficients and the pressure distribution for a cylinder array are greatly influenced by the spacing among the cylinders and their arrangements, in addition to the Reynolds number. Hereafter, the spacing ratio is defined as the ratio of center-to-center distance between two adjacent cylinders to the cylinder diameter.

The experimental study on flow interference of two cylinders in a side-by-side arrangement by Bearman and Wadcock (1973) showed the presence of a mean repulsive force between the cylinders in close proximity (spacing ratio up to 2). They also observed two vortex streets when the gap was greater than one diameter, but only one street of alternate vortex shedding when the cylinders were very close to each other.

Sayers (1988, 1990) carried out a series of experimental studies on flow past four equal-spaced cylinders in square arrangements with spacing ratio ranging from 1.1 to 5 at \( Re=3\times10^4 \) and concluded that vortex shedding frequency was largely influenced by the spacing ratio and the incoming flow direction. When the spacing ratio was larger than 4, the Strouhal number of each cylinder was equal to that of a single cylinder, indicating the weak interference among the cylinders at large spacing ratios. The flow pattern and the force coefficients were also strongly influenced by the orientation of the cylinder array. At small spacing ratios (1.1~3), little change in the direction of the free stream velocity may lead to significant changes in vortex shedding frequency and force coefficients.

Farrant et al. (2000) numerically investigated the laminar 2-D flow past four equal-spaced cylinders at \( Re=200 \) in two orientations, i.e., the in-line alignment and the alignment angle of 45° using a cell boundary element method. Two spacing ratios, namely, 3 and 5 were considered and it was found that, in the in-line arrangement, the in-phase vortex-shedding mode occurred when the spacing ratio was 3, while anti-phase vortex-shedding mode dominated at the spacing ratio of 5.

By conducting flow visualization studies, Lam et al. (2003a) and Lam and Lo (1992) found suppression of vortex shedding from the upstream cylinders at spacing ratios less than 3.94, which was defined as the critical spacing ratio for flow pattern transformation. Lam and Fang (1995), Lam et al. (2003b) and Lam and Zou (2007) further studied the effects of spacing ratio on the pressure distributions and the lift and
drag coefficients at sub-critical Reynolds numbers. Numerical studies by Lam et al. (2008) and Lam et al. (2003a) were focused on the prediction of the vortex shedding flow pattern based on 2-D studies. They found that 2-D simulations cannot adequately represent flow around four cylinders and there was a large discrepancy between experimental measurements and 2-D numerical results of the critical spacing ratios, which have been remedied and well captured by the 3-D simulations later by Lam and Zou (2010).

When two or more cylinders are placed close to each other, the proximity among them results in complex interactions between the shear layers and Kármán vortex streets. A distinct flow feature for multiple cylinders is the anti-phase and in-phase vortex shedding, which have been found in the wake of a pair of side-by-side cylinders at spacing ratios beyond 2.0 (Sumner, 2010). The in-phase vortex shedding occurs when each cylinder in the pair synchronously forms vortices of the same rotational sign, while the anti-phase occurs when these vortices are in the opposite direction. As a result, the wake comprises two parallel vortex streets in either the in-phase or the anti-phase form. It has also been found that the in-phase vortex shedding may lead to the eventual formation of a binary-vortex street, as reported by Williamson (1985). It is generally believed that the vortex shedding is predominantly synchronized in an anti-phase pattern, but the predominance of the anti-phase shedding synchronization decreases with the increase of spacing ratio (Alam et al., 2003, Sumner, 2010). The two synchronized vortex shedding modes can remain stable for many cycles, although the flow is capable of changing modes at any time. The reason for this change in modes is not clear yet. However both Williamson (1985) and Farrant et al. (2000) attributed this phenomenon partly to the perturbations arising from the experiments.

The 3-D flow characteristics are much more common in real life engineering applications but are difficult to capture experimentally. Direct numerical simulations (DNS) have become a powerful tool to study fundamental flow structures of 3-D flow at relatively low Reynolds numbers although it is still challenging at large Reynolds numbers due to the tremendous computational time requirement. As discovered by Williamson (1985), turbulent flows in the subcritical regime show some similar characteristics to those at low Re numbers. Given the fast growth of computational power in recent years, 3-D DNS has been used to investigate flow features around a cylinder at low Reynolds numbers in the turbulent regime, for instance, by Barkley and
This paper presents a 3-D direct numerical simulation (DNS) of flow around four equal-spaced cylinders in an in-line square arrangement with spacing ratio of 2 and Re in the range from 100 to 500. The constant spacing ratio of 2 is investigated for two reasons: (1) the distance between two column structures is around two times of the column diameter in many engineering applications, and (2) such a small spacing ratio is not extensively investigated in previous studies. The Navier-Stokes equations are solved using OpenFOAM® at a range of Reynolds numbers. The focus of this study is to examine the effects of the Reynolds number on the vortex shedding flow. Wake transitions are classified based on flow characteristics, vortex shedding patterns and force coefficients.

4.2 Numerical method

4.2.1 Computational models

The vector form of the 3-D Navier-Stokes (NS) equations for incompressible flows can be expressed as

$$\frac{\partial \hat{U}}{\partial t} + \nabla \cdot (\hat{U} \hat{U}) - \nu \nabla^2 \hat{U} = -\frac{\nabla p}{\rho}$$

(4-1)

$$\nabla \cdot \hat{U} = 0$$

(4-2)

where the velocity vector $\hat{U}$ has three components $U_x$, $U_y$, and $U_z$ in the Cartesian coordinates $(x, y, z)$, respectively; $\rho$ is the density of the fluid, $\nu$ the fluid kinematic viscosity, $t$ is the time, and $p$ is the pressure. The NS equations are solved by the Open source Field Operation and Manipulation (OpenFOAM®) C++ libraries, which is a free source CFD package developed by OpenCFD Ltd. The standard solver icoFoam provided by OpenFOAM® is used in this paper. The finite volume method is applied in the solver and the pressure-velocity coupling is dealt with by the Pressure Implicit with Splitting of Operators (PISO). Discretization schemes for terms of the NS equations can be chosen from a number of methods provided in OpenFOAM®. The convection terms are discretized using the Gauss cubic scheme, while the Laplacian term and pressure term in the momentum equations are discretized using the Gauss linear scheme. The Euler implicit scheme is adopted for the temporal discretization. Details of these
schemes can be found in the source code of OpenFOAM®.

4.2.2 Boundary conditions

A rectangular computational domain of \((30+L)\times(20+L)\times9.6D\) is chosen in this study with \(L\) being the spacing ratio, as defined in Figure 4-1. Only one spacing ratio, i.e. 2, is considered in the current study with \(Re\) ranging from 100 to 500. The domain size in the axial direction of the cylinders is \(9.6D\), which is the same as that used in Zhao et al. (2009). The two upstream cylinders are labeled as 1 and 2, while the two downstream ones are labeled as 3 and 4, respectively, as illustrated in Figure 4-1 (a) and (b). The inflow boundary is \(10D\) upstream of cylinders 1 and 2. The outflow boundary is \(20D\) downstream of cylinders 3 and 4 and two sides boundaries are \(10D\) away from the cylinders. The distances for inlet and side boundaries were found to be sufficient in the study of Lam and Zou (2010) and the outlet boundary location is chosen based on the study by Lei (2000).

![Figure 4-1](image)

Figure 4-1 The schematics of the computational domain for four circular cylinders in an in-line square configuration and the grids distribution around the cylinders. (a) Top view of computational domain; (b) front view of the computational domain; (c) grid distribution around the four circular cylinders. The inlet flow direction is from left to right.

The initial values for the velocity and the pressure in the whole domain are set to zero and the boundary conditions for the governing equations are: (i) at the inlet, a uniform velocity in the \(x\)-direction is given and the Neumann condition is applied for pressure; (ii) at the outlet, the pressure is set to be zero and the velocity gradients in the streamwise direction are zero; (iii) symmetry boundary conditions are applied at the two lateral boundaries and at the two end boundaries that are perpendicular to the cylinders;
(iv) no-slip boundary condition is adopted along the cylinder surfaces.

4.3 Grid dependence tests and model validations

A two-dimensional (2-D) structured mesh is generated with quadrilateral elements in the $x$-$y$ plan and high mesh resolution near the surface of the cylinders. The 2-D mesh is then extended in equidistance in the $z$-direction of certain layers to generate the final 3-D mesh. Figure 4-1 (c) shows the mesh distribution around the four cylinders. The grid dependence study was carried out at the highest Reynolds number used in this study, i.e. $Re = 500$. At $Re=500$, the flow around a single cylinder is fully three-dimensional. Three meshes with different resolutions, i.e. the coarse, medium and fine meshes, are tested. The difference between the medium and fine meshes is the near wall mesh density in the radial direction of the cylinder, which can be evaluated by the non-dimensional distance of the first nodal point to the wall, $y^+ = \mu_j \Delta / \nu$, where $\Delta$ is the distance from the wall and $\mu_j$ is the friction velocity. The $y^+$ values in this study are kept below 1.0 and those for the medium and fine meshes are 0.424 and 0.055, respectively. Table 4-1 shows the parameters of the three meshes and the resulting mean drag coefficients $\overline{C_D}$, root-mean-square (RMS) lift coefficients $C'_L$ and Strouhal numbers $St$. The force coefficients and the Strouhal number are defined as

$$
C_D = \frac{F_x}{(\rho U^2 DH / 2)}, \quad C_L = \frac{F_y}{(\rho U^2 DH / 2)}, \quad St = \frac{f_s}{D / U},
$$

where $F_x$ is the drag force in the $x$-direction, $F_y$ is the lift force in the $y$-direction, $H$ is the cylinder length, and $f_s$ is the frequency of the fluctuating lift force. The $F_x$ and $F_y$ are obtained by integrating the pressure and shear stress along the whole cylinder surface. It can be seen that the differences in $\overline{C_D}$, $C'_L$ and $St$ between the results from the medium mesh and the fine mesh are less than 1%, 4% and 1%, respectively. Values from Batcho and Karniadakis(1991), Lu and Ling(2002) and Mittal and Balachandar(1995) are also included in Table 4-1 for comparison. The $C_D$ value from the medium mesh is 1.224, which falls in the range of previous research results between 1.07 and 1.24. The Strouhal numbers $St$ of 0.21 also agrees well with other results. Since the Reynolds numbers investigated in this study are all smaller or equal to 500, it is expected that the medium mesh listed in Table 4-1 is adequate for all the simulations carried out in this study.
In order to further validate the numerical model, vortex shedding flow from a single cylinder is simulated using the medium mesh at Reynolds numbers ranging from 100 to 500. According to Williamson (Williamson, 1988) and Zhao et al. (2013), the transition of the flow from 2-D to 3-D has significant influence on the vortex shedding frequency. Williamson (Williamson, 1988) observed two discontinuities in the Strouhal number as the wake flow transits from 2-D to 3-D. The first discontinuity occurs as the flow changes from 2-D to mode A (Re = 180–194), which is characterized by the inception of vortex loops and the deformation of streamwise vortex pairs and the second discontinuity occurs as the flow changes from mode A to mode B (Re = 230–250), which is characterized by finer-scale streamwise vortices with spanwise length scale of about one diameter.

The variations of Strouhal numbers $St$ are compared with the experimental results of Williamson (Williamson, 1988), Prasad and Williamson (Prasad and Williamson, 1997) and numerical results from Zhao et al. (2013) in Figure 4-2. The numerical results agree well with the experimental data except at Reynolds number around 200. The first discontinuity in the present study occurs after Re = 200, which is larger than that in the experimental result. The 3-D flow occurs later than what was observed in the experiments because the symmetric end condition is used in the present study, as pointed out by Zhao et al. (2013). Miller and Williamson (1994) found that the laminar regime for parallel shedding can be extended up to Re=194 and even beyond 200 if there was no effect from the end condition. The symmetric boundary condition used in this study produces little end effect on the solution.

<table>
<thead>
<tr>
<th>Mesh density and selected published studies</th>
<th>Nodes circumstance/length</th>
<th>Node number</th>
<th>$y^+$</th>
<th>$\Delta$</th>
<th>$\overline{c}_w$</th>
<th>$C_L$</th>
<th>$S_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse mesh</td>
<td>80/50</td>
<td>671,004</td>
<td>0.514</td>
<td>0.009</td>
<td>1.231</td>
<td>0.368</td>
<td>0.210</td>
</tr>
<tr>
<td>Medium mesh</td>
<td>100/96</td>
<td>995,200</td>
<td>0.424</td>
<td>0.007</td>
<td>1.224</td>
<td>0.356</td>
<td>0.211</td>
</tr>
<tr>
<td>Fine mesh</td>
<td>100/96</td>
<td>1,121,320</td>
<td>0.055</td>
<td>0.0009</td>
<td>1.213</td>
<td>0.342</td>
<td>0.213</td>
</tr>
<tr>
<td>Batcho and</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.16</td>
<td>-</td>
<td>0.2$^*$</td>
</tr>
<tr>
<td>Mittal and</td>
<td>-</td>
<td>518,400</td>
<td>-</td>
<td>-</td>
<td>1.24$^a$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lu and Ling (2002)</td>
<td>-</td>
<td>1,081,665</td>
<td>-</td>
<td>-</td>
<td>1.07</td>
<td>0.3$^b$</td>
<td>0.202</td>
</tr>
</tbody>
</table>

Table 4-1 Mesh dependence check and model validation for a single cylinder at Re = 500.

$^a$ Values at Re=525

$^b$ Values were extracted from graphs in the corresponding reference paper
Figure 4-2 Variations of Strouhal number with Reynolds number, compared comparison between the present results with experimental data from Williamson (Williamson, 1988) and Prasad and Williamson (Prasad and Williamson, 1997) and numerical results from Zhao (Zhao et al., 2013).

Figure 4-3 shows the calculated mean drag coefficients $C_D$ and RMS lift coefficients $C'_L$ versus Reynolds number over the laminar and wake transition regimes, together with the numerical results by Zhao et al. (Zhao et al., 2013) and measurements by Wieselsberger (1921), which was extracted from Schlichting and Gersten (2003). The agreements between the present results on $C_D$ and $C'_L$ and those by Zhao et al. (Zhao et al., 2013) and Schlichting and Gersten (2003) are good, especially at the high end of the Reynolds number investigated. While two numerical models slightly under predicted the drag coefficient compared with Wieselsberger (1921) at Re below 150, the RMS lift coefficient predicted in this study are slightly smaller than those from Zhao et al. (2013) in the 2-D regime. It appears that the discontinuity that occurs in the St $\sim$ Re curve due to the flow transition also occurs in the $C_D$ and $C'_L$ curves. The RMS lift coefficient $C'_L$ increases in the laminar regime from 0.23 at Re = 100 to 0.47 at Re = 200, then it reduces suddenly to 0.36 after the wake flow transits to mode A. Another sudden increase in $C'_L$ occurs as the wake changes from mode A to mode B at Re = 260. In mode B (Re>260), $C'_L$ decreases smoothly with the increase of Re. Similarly, the drag coefficient $C_D$ gradually decreases with the increase in Re except a sudden decrease at Re=200 and a sudden increase at around Re = 260.
Figure 4-3 Variation of force coefficients with Reynolds number compared with numerical study (Zhao et al., 2013) and measurements by Wieselsberger (1921) which were taken from Schlichting and Gersten (Schlichting and Gersten, 2003).

(a) $Re = 245$

(b) $Re = 255$

Figure 4-4 Three-dimensional views of the results for two tandem cylinders at (a) $Re = 245$ and (b) $Re = 255$. Translucent surfaces represent iso-surfaces of $|\omega_z|=1.5$. Solid blue and red surfaces represent iso-surfaces of negative and positive $|\omega_x|=0.04$, respectively. Solid dark grey represent the two cylinders with spacing ratio $L/D = 2.3$. Flow direction is from left to right.

The present numerical model is further validated against flow around two tandem circular cylinders where a linear instability analysis shows that the three dimensional wake transition for two tandem cylinders with spacing ratio $L/D = 2.3$ occurs at $Re=250$ (Carmo et al., 2010). Tests are carried out at $Re=245$ and 255 for the two tandem cylinders in order to capture the flow transition using the present numerical model. The medium mesh size used in the mesh dependence study is used in this simulation. Figure 4-4 shows iso-surfaces of vorticity at $Re=245$ and 255, where the vorticity components are defined as
\[ \omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \quad \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \quad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \] (4-4, a, b, c)

Figure 4-4 (a) and (b) display the vorticity iso-surfaces at roughly the same phase of the shedding cycle. It can be seen that the \( \omega_z \) are similar in both cases. The difference is that \( \omega_x \) is hardly seen at \( Re=245 \), indicating that it is a two-dimensional wake, while the three-dimensional structures are clearly observed at \( Re=255 \) in Figure 4-4 (b). This result is consistent with the prediction of the linear stability analysis by Carmo et al. (Carmo et al., 2010).

To check the influence of blockage ratio, the computational domain is doubled to be \( 40D \times 62D \) in \( xy \) plane for one additional cases at \( Re = 100 \) for the four cylinders in a square configuration. It is found in Figure 4-5 that no obvious change to the flow field occurs as shown by the vorticity distribution and iso-surface of \( \lambda_2 \), which is defined in section 4.4.1. Thus the numerical results are independent from the size of the computational domain. The slightly difference in Figure 4-5 is due to the position of dislocation, which intermittently changes with space and time.

![Figure 4-5 Comparison of the three-dimensional results at Re=100 for four cylinders at computational domain of (a) \( xy = 40D \times 62D \) and (b) \( xy = 20D \times 32D \). Yellow surfaces represent iso-surfaces of \( \lambda_2 = -0.4 \), which is defined in section IV A. Solid blue and red surfaces represent iso-surfaces of negative and positive \( |\omega_z| = 0.4 \), respectively. Solid dark grey represent the four cylinders with spacing ratio \( L/D = 2.0 \).](image)

### 4.4 Numerical results

The numerical simulations of flow around four cylinders in a square configuration are carried out at Reynolds numbers ranging from \( Re = 100 \) to 500, with an interval of 20 for \( Re \) exceeding 180 in order to capture the wake transition. The simulations are conducted for non-dimensional time of at least \( t^* = 650 \) in order to obtain fully developed wake flows, where the non-dimensional time is defined as \( t^* = Ut/D \).
Observations in the present study show that the wake flow is usually fully developed after about $t^* = 250$.

Figure 4-6 Instant iso-surfaces of streamwise vorticity $\omega_x$ and $\lambda_2$ at different Reynolds numbers. The red and blue colors denote positive and negative iso-surfaces of $|\omega_x|=1$, respectively. Translucent yellow color represents iso-surfaces of $\lambda_2 = -1$. Only at Re=100, $|\omega_x|=0.4$ and $\lambda_2 = -0.4$ is shown because of the weak three-dimensionality. The flow direction is from upper left to lower right.

4.4.1 Wake flow patterns

In this section, wake flow patterns are investigated by examining the vortex shedding, time history of force coefficients and formation length of the wake.
The vorticity and the second negative eigenvalue $\lambda_2$ of the tensor $\Psi^2+\Omega^2$ are two most popularly used methods for identifying the wake flow patterns and describing the vortex flow motion around structures. Here $\Psi$ and $\Omega$ are the symmetric and the anti-symmetric parts of the velocity-gradient tensor, respectively. Jeong and Hussain (Jeong and Hussain, 1995) demonstrated that the second eigenvalue $\lambda_2$ is able to accurately identify the location of the vortex cores, by capturing the minimum pressure in a plane perpendicular to the vortex axis at both high and low Reynolds numbers. Figure 4-6 shows the instant iso-surfaces of $\lambda_2$ and iso-surfaces of streamwise vortices ($\omega_x$) in the wake of the cylinder array, respectively, at different Re and $t^*=650$.

Based on observations on the distribution of $\lambda_2$ and $\omega_x$, the flow around the four cylinder array can be classified into four regimes, hereafter referred to as regime 1 to 4. A typical characteristic of the wake flow regime 1 at $100 \leq \text{Re} \leq 220$ is the inclination of the spanwise vortices, which can be identified in Figure 4-6 (a) - (d), where $\lambda_2$ in the wake of the cylinders are unevenly distributed along the axial direction of the cylinder and the streamwise vorticity is generally weak except at locations where the spanwise vortices bend or disconnect. The inclination of the spanwise vortices is due to the phase variation of the vortex shedding along the cylinder span. The strength of the spanwise vortices increases with the increase in Reynolds number. One interesting flow characteristics observed in regime 1 is that three-dimensionality of the wake behind the four cylinder array develops at smaller Re than the critical Re of about 190 for a single cylinder in regime 1. The physical mechanism responsible for this early transition will be explained in the next section.

The wake flow regime 2 between $240 \leq \text{Re} \leq 300$ is characterized by regular wavy spanwise vortices and rib-shaped streamwise vortices. At Re = 240, as shown in Figure 4-6 (e), a row of finer rib-like streamwise vortices are generated and shed in the wake of the cylinder array and the wavy spanwise vortices are stronger than those at Re<220. Eight pairs of $\omega_x$ are observed in Figure 4-6 (e), which corresponds to eight waves in the wavy spanwise vortex structure, resulting in a wavelength of approximately 1.2. The spanwise wavelength stays unchanged as Reynolds number is increased from 240 to 300. This dominant wavelength is very close to that in mode B vortex shedding around a single circular cylinder(Williamson, 1996b, Williamson, 1988). Although the spanwise vortices are wavy for $240 \leq \text{Re} \leq 300$, they are aligned almost perfectly parallel to the cylinders. The flow between the upstream and the downstream cylinders
are still 2-D for $240 \leq \text{Re} \leq 300$ since the vortex tubes between the cylinders are exactly parallel to the cylinders and no streamwise vortices are observed among the cylinders.

Irregular iso-surfaces of $\lambda_2$ and $\omega_x$ develop when Reynolds number exceeds 320, as shown in Figure 4-6 (i). In regime 3 between $320 \leq \text{Re} \leq 380$, the streamwise vortices are still generally in pairs. However, the iso-surfaces of the vorticity are not as regular as those in regime 2 of $240 \leq \text{Re} \leq 300$. It is difficult to quantify the wavelength in the spanwise direction through visual observations. The inclined spanwise vortices start to appear in this range of Reynolds number, indicating the phase difference of vortex shedding in the spanwise direction. The vortex tubes of $\lambda_2$ between the upstream and downstream cylinders also start to bend as shown in Figure 4-6 (i)-(l). However, three dimensionality of the flow between the upstream and downstream cylinders is still too weak to be detected by the iso-surfaces of $\omega_x = \pm 1$.

Regime 4 is found at Re larger than 400. It seems that Re=400 is a critical Reynolds number, beyond which the flow between the upstream and the downstream cylinders becomes three-dimensional. Between the upstream and downstream cylinders, the spanwise vortices become wavy and streamwise vortices with larger spanwise wavelengths start to appear in Figure 4-6 (m) at Re = 400. Strong streamwise vortices downstream the two downstream cylinders occupy the space very close to the cylinder surface. As Re exceeds 420, the streamwise instability in the gap between the up- and downstream cylinders disturbs the spanwise vortices and makes them wavy in the spanwise direction. It appears that the spanwise vortices in the wake of the cylinders are still stronger than the streamwise ones at $400 \leq \text{Re} \leq 500$ based on Figure 4-6.

The time histories of the force coefficients are discussed to illustrate the vortex shedding flow around the four cylinders. Figure 4-7 shows the time histories of lift coefficient for cylinders 1 and 4 at four selected Re, each one representing one flow regime. The force coefficients on cylinders 2 and 3 are similar to those on cylinders 1 and 4, respectively. Variations of the amplitude of the force coefficients with time on the upstream cylinder 1 are weak and regular for Re<400, indicating feeble three dimensionality of the flow around upstream cylinders as shown in Figure 4-6. Relative strong irregularity of the force coefficients on the upstream cylinder 1 with time for Re$\geq$400 are caused by the three dimensional vortex tubes appearing between the upstream and downstream cylinders as shown in Figure 4-6. The irregularities of the force coefficients on the downstream cylinder 4 with time show strong correlation.
between the force coefficient and the wake features behind the cylinder array.

![Figure 4-7 Time histories of lift force coefficient on the upstream cylinder 1 (dashed lines) and the downstream cylinder 4 (solid lines) for one selected Reynolds number from each flow regime. To facilitate comparison, forces are translated in the vertical axis, where the quantity between any two neighboring ticks is 1.](image)

For $100 \leq Re \leq 220$, the irregularity of the lift coefficient with time is strong but the amplitudes of the lift coefficient are relatively small. These are likely caused by the weak three dimensionality of the wake and the phase variations of vortex shedding along the spanwise direction of the cylinder. For $240 \leq Re \leq 300$, the irregularity of the lift coefficient with time is weak but the amplitudes of the lift coefficient are relatively large. This is in accordance with rib-shaped vortex shedding at this Re range. The beating effect of the force coefficients is found at $320 \leq Re \leq 380$, but in relatively small amplitude. This is due to the irregular vortex shedding along the spanwise directions and the vortex dislocations which will be discussed later on. For Re>400, the force coefficients are characterized by strong irregularity and strong beating effect. This again reflects the fully turbulent wakes for downstream cylinders.

The wake structure behind the downstream cylinders is further investigated by examining the formation length of the wake. The formation length $L_w$ of a wake, physically, is a measure of the dimension of the recirculation zones formed by the time-
The averaged mean velocity behind downstream cylinders (Silva et al., 2003, Williamson, 1996a). The formation length in the present study is determined in a similar way that used in Silva et al. (2003) by plotting the mean velocity $U_x$ along $y=2$ that passes through the center of cylinder 4 on the middle-section ($z=4.8D$). The distance between the two zero-crossing points behind cylinder 4, as shown in Figure 4-8, is defined as the formation length, denoted by $L_w$. The variation of formation length with Reynolds number is shown in Figure 4-9. It is found that the formation length decreases almost linearly with Re for $100 \leq Re < 200$, $200 < Re < 240$ and $240 < Re < 300$ but at different rates. The decreasing rate of $L_w$ with Re changes at Re=180-200 where three dimensionality of the flow starts to develop. It changes again at Re=240 where the regular wavy spanwise vortices and rib-shaped streamwise vortices develop. Sharp changes of $L_w$ are observed at Re = 300~320, where severe vortex dislocations in the wake of the downstream cylinders start to occur and at Re = 400~420, where the three dimensionality of the wake between the upstream and downstream cylinders develops. The formation length decreases from $2.5D$ at Re=100 to $0.75D$ at Re=300. It increases back to $1.2D$ in the Reynolds number range of $320 \leq Re \leq 380$ and decreases to about $0.7D$ for Re $> 420$. The discontinuity of $L_w$ at Re = 320-400 agrees with the observation by Carmo and Meneghini(2006), where the formation length of the vortex shedding for two cylinders in tandem is found to increase at wake transition region. The lift force amplitude also reduces as the formation length increases as shown in Figure 4-7 in regime 3.
Figure 4-8 The definitions of the formation length based on: a) the streamlines, and b) by the velocity component $U_x$ as a function of $x$ (case from $\text{Re}=320$).

Figure 4-9 The length of the formation length $L_w$ as a function of $\text{Re}$.

Based on the above observations, the vortex shedding flow around the 4 cylinders can be classified into four regimes over $100 \leq \text{Re} \leq 500$. Regime 1 ($100 \leq \text{Re} \leq 220$) is characterized by very weak and inclined spanwise vortices. The streamwise vortices in regime 1 are very weak except at the locations where the spanwise vortices bend. Regime 2 ($240 \leq \text{Re} \leq 300$) is characterized by the regular wavy spanwise vortices and rib-shaped streamwise vortices. The wake flow in regime 2 is similar to the mode B in the single cylinder case. The spanwise vortices are parallel to the cylinders in regime 2. Regime 3 ($320 \leq \text{Re} \leq 380$) is characterized by extended formation length, inclined spanwise vortices and smaller force coefficients. Regime 4 ($400 \leq \text{Re} \leq 500$) is
characterized by the occurrence of three-dimensionality between the upstream and downstream cylinders and the ceasing of the vortex dislocation.

4.4.2 Physical mechanisms of the flow regimes

Physical mechanisms responsible for the flow regimes identified in the previous section are explored in this section.

The primary reason for the distinct flow features in four regimes and regime transitions is due to the shear layer interactions around the four cylinders. To explain this, spanwise vorticity contours at the middle cross section of the cylinder array for one typical Re from each regime are shown in Figure 4-10. In regime 1 (100 ≤ Re ≤ 220) the four cylinders largely behave as a single structure, as shaded by a translucent square cylinder in Figure 4-10 (a), which has an effective length of 3D. It is observed in Figure 4-10 (a) that shear layers at the inner sides of the two tandem cylinder pairs are weak and quickly merged with the shear layers of the same signs from outer sides of the two tandem cylinder pairs, resulting in a single Kármán vortex street behind the four cylinder array. An effective Reynolds number based on the effective length of the translucent square cylinder is about three times of the Re based on a single cylinder, leading to an effective Re range of 300 to 660 in wake regime 1. This explains why the onset of three-dimensionality in the wake of the four cylinder array occurs at smaller Re than the critical Re (about 190) for 3-D wake transition of a single cylinder.

With the increase of Re in regime 2 (240 ≤ Re ≤ 300), however, the shear layer from the inner side of each tandem cylinder pair becomes strong enough to interact with the shear layer from the outer side of the same tandem cylinder pair, resulting in two distinct Kármán vortex streets behind the cylinder array. As seen from Figure 4-10 (b), in this wake regime, the four cylinders behave as two structures in a side-by-side configuration, each side comprising two tandem cylinders. The two Kármán vortex streets are synchronized in a stable anti-phase mode, similar to one of the flow modes for two side-by-side cylinders discovered in previous independent studies(Williamson, 1985). The stable anti-phase synchronization of the two Kármán vortex streets is believed to be responsible for the regular wavy spanwise vortices and rib-shaped streamwise vortices in regime 2.

Similar to regime 2, two Kármán vortex streets exist in regime 3 too. However the two Kármán vortex streets in regime 3 are mostly synchronized in an in-phase mode
(See Figure 4-10, c), only intermittently switching to an anti-phase synchronization. The in-phase synchronization observed in regime 3 appears to be less stable than the anti-phase synchronization observed in regime 2. The interaction between the two Kármán vortex streets in regime 3 is clearly stronger than that in regime 2. This is believed to be one of the major reasons for the severe vortex distortions observed in regime 3.

There are four interacting wakes in regime 4. Vortex shedding occurs behind all four cylinders in regime 4, as indicated by two arrows in Figure 4-10 (d). The strong interactions between the four wakes lead to many fine-scale vortices in the wake behind the four cylinder array. Regime 3 can be regarded as a transition regime from interactions between two wakes in regime 2 to four wakes in regime 4.

The transition from regime 1 to regime 2 behind the four cylinder array is believed to be induced by onset of three-dimensionality behind the rear cylinder in each tandem pair of cylinders in the array. The transition from regime 1 to regime 2 behind the four cylinder array shares many common features with the wake transition from mode A to mode B of a single cylinder, and with the fundamental modes of the wake transition to turbulence for two tandem cylinders (named modes T1, T2 and T3)(Carmo et al., 2010). The manifestation of mode B (or T1 for two tandem cylinder case) is clearly seen in regime 2 and thus is believed to be the mechanism of flow transition for four cylinders from regime 1 and regime 2. With the increase of Re, the three-dimensionality further
contributes to other wake transitions. To explore this transition mechanism, mean standard deviation $U_{z}^{\text{DEV}}$ (in space) of the spanwise velocity at two locations in the wake is investigated in Figure 4-11. The $U_{z}^{\text{DEV}}$ is defined as,

$$U_{z}^{\text{DEV}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (U_{z}^i - \overline{U}_z)^2}$$

(4-5)

where $\{U_{z}^1, U_{z}^2, \ldots, U_{z}^N\}$ are the observed values of $U_{z}$ at an instant and $\overline{U}_z$ is the mean value of these observations, while the denominator $N$ stands for the size of the sample. A total of 100 points were sampled at each position of $(x, y) = (0, -1)$ and $(2, -1)$ along the spanwise length ($9.6D$). The two positions correspond to $1D$ downstream cylinder 1 and cylinder 4, respectively (refer to Figure 4-1). The data shown in Figure 4-11 are averaged over time for at least 200 non-dimensional time after the calculation has reached to a dynamic equilibrium. For position $(x, y) = (0, -1)$ downstream cylinder 1, $U_{z}^{\text{DEV}}$ along the spanwise direction is less than 1% of the inlet velocity in regime 1, 2 and 3, then it exponentially raises to about 3.5% at Re=400~420 and further to about 6% at larger Re in regime 4. The sudden increase in $U_{z}^{\text{DEV}}$ at $(x, y) = (0, -1)$ corresponds to the flow transition from regime 3 to regime 4. It is interesting to notice that the $U_{z}^{\text{DEV}}$ is larger at regime 1 ($100 \leq \text{Re} \leq 220$) when the inclined spanwise vortices dominate the wake than that at regime 2 ($300 \leq \text{Re} \leq 300$) when the rib-shaped streamwise vortices occur. Similar feature is observed at position $(x, y) = (2, -1)$ downstream cylinder 4 but with larger $U_{z}^{\text{DEV}}$ values than those at $(x, y) = (0, -1)$. The $U_{z}^{\text{DEV}}$ at position $(x, y) = (2, -1)$ is about 3.5% of the inlet velocity at Re=240 which corresponds to the start of regime 2 in Figure 4-6 (e). This value is roughly the same as the value of $U_{z}^{\text{DEV}}$ downstream cylinder 1 at Re=400, suggesting that wake flow downstream cylinder 1 at Re=400 is similar to that downstream cylinder 4 at Re=240. The 3-D wake feature starts to appear in the confined space between the up- and downstream cylinders at Re=400, while well-organised three dimensional wakes develop behind the two downstream cylinders at Re=240.
Figure 4-11 Variation of standard deviation of the velocity $U_z^{DEV}$ (in spanwise space) with the Reynolds number. Data plotted here are in the positions of $(x, y) = (0, -1)$ and $(2, -1)$ respectively and averaged over time.

Finally the phase difference of vortex shedding at different locations along the spanwise direction is believed to be another cause for the vortex dislocations observed in regime 1 and regime 3. Williamson (Williamson, 1992) found the ‘spot-like’ vortex dislocations grow after a circular cylinder when three dimensional vortex shedding takes place, along with the two fundamental modes (mode A and mode B). This was attributed to the observation that the primary Kármán vortex shedding moves out of phase in the wake along the spanwise of the cylinder. It is found in the present study that the vortex dislocations dominate in regime 1 and regime 3 with large scale flow structures and, the vortex dislocations intermittently change with space and time. This is
consistent with the findings from previous independent studies (Williamson, 1992). Figure 4-12 shows the contours of the spanwise vorticity at different cross-sections for \( \text{Re}=180 \) and \( t^*=650 \). It can be clearly seen that the vortex shedding at different \( z \)-locations is not synchronized with each other. The phase difference in vortex shedding along the spanwise direction is believed to be the cause for vortex dislocations and the inclined spanwise vortex tubes as shown in Figure 4-6 (b). It is found that vortex shedding flow varies significantly along the spanwise direction of the cylinders over \( 100 \leq \text{Re} \leq 220 \) (regime 1) and over \( 320 \leq \text{Re} \leq 380 \) (regime 3), while over other cases, vortex shedding at different locations along the cylinders appears to be more synchronized, as illustrated in Figure 4-13.

![Figure 4-12](image)

Figure 4-12 Instantaneous spanwise vorticity contours illustrating the cessation of dislocation along the spanwise direction at \( \text{Re}=300 \) and \( t^*=650 \). Contours vary from \( \omega_z = -1 \) (cold blue colors & dashed lines) to \( \omega_z = 1.0 \) (warm red colors & solid lines)

4.4.3 **Force coefficients**

When the four cylinders are arranged close to each other, the repulsive and attractive forces can be found among them, as pointed out by Bearman and Wadcock (1973) and Farrant et al. (2000). Figure 4-14 shows the time histories of the force coefficient at \( \text{Re}=280 \) (regime 2) and \( \text{Re}=360 \) (regime 3). In Figure 4-14 (a), where the time histories of forces are very regular and periodic, the lift forces on cylinders 1 and 3 are in anti-phase with those on cylinders 2 and 4, respectively, resulting in profound periodic repulsive and attractive forces between cylinders 1 and 2 and between cylinders 3 and 4. The periodic repulsive and attractive forces between the two downstream cylinders have
higher amplitude than those between the two upstream cylinders. At Re = 280, the lift forces are exactly symmetric, resulting in zero mean lift force on the whole system. The anti-phase vortex shedding mode in Figure 4-14 (a) agrees with that found by Lam et al. (2008) at Re = 200. Unlike those in Figure 4-14 (a), the lift forces on cylinder 3 and 4 in Figure 4-14 (c) are very irregular and are in anti-phase with each other only occasionally. It is interesting to see that whenever the lift amplitude on cylinder 3 is large, the lift amplitude on cylinder 4 is small or vice versa. The irregularity of the lift coefficient at Re = 360 is likely due to the three dimensionality of the wake structures behind cylinders 3 and 4. The drag coefficients on the two upstream or the two downstream cylinders are identical at Re = 280. This is no longer the case for flow with Re=360, which is likely due to the three dimensionality of the wake structures behind cylinders 3 and 4 again.

![Figure 4-14 Time series of the lift and drag coefficients of the cylinders at Re=280 and Re=360.](image)

Phase angle difference between the lift coefficients on two downstream cylinders is quantified for all the cases studied in this paper. The variation of the phase angle difference with Reynolds number is shown in Figure 4-15. The in-phase, anti-phase and bi-stable vortex shedding modes can be inferred by examining the results shown in Figure 4-15, where the phase angles of 0° and 180° correspond to in-phase and anti-phase vortex shedding, respectively. It can be seen that the wake is dominated by anti-phase vortex shedding for most of the Re investigated, except at regime 3 (320 ≤ Re ≤ 380) and at Re=100, where the in-phase vortex shedding prevails, while at Re=180 and Re=400 the wake is in a bi-stable state. This corresponds well with the observations in the spanwise vorticity plots as shown in Figure 4-12 and Figure 4-13. It
is worth noting that the phase angle difference shown in Figure 4-15 is statistically derived from the integrated lift forces on cylinder 3 and cylinder 4. The anti-phase and in-phase vortex shedding modes do occur alternatively in time, for instance in regime 4, and also coexist at certain specific time along the spanwise direction of the cylinders. It has been reported that vortex shedding mode is also dependent on spacing ratio for two cylinders in a side-by-side arrangement (Williamson, 1985, Sumner, 2010) and for a four-cylinder array in a square arrangement (Lam et al., 2008, Farrant et al., 2000).

The variations of the mean drag coefficient \( \overline{C_D} \) and the RMS lift coefficient \( C_L' \) on cylinders 1 and 4 with Reynolds number are shown in Figure 4-16. They are based on the fully-developed numerical results in at least 200 non-dimensional time. Because \( \overline{C_D} \) and \( C_L' \) on cylinders 2 and 3 are similar to those on cylinders 1 and 4, respectively, they are not included in Figure 4-16. Flow regime changes can be clearly identified from the RMS lift coefficient \( C_L' \) of cylinder 4. The mean drag coefficient \( \overline{C_D} \) on cylinder 1 is very close to that of a single cylinder, while \( \overline{C_D} \) on cylinder 4 is reduced dramatically because it is in the wake of the cylinder 1 (see Figure 4-16, a). The RMS lift coefficient on cylinder 4 is greater than that on cylinder 1 due to the vortex shedding from the downstream cylinders. The RMS lift coefficient of cylinder 4 in regime 1 (\( 100 \leq Re \leq 220 \)) is smaller than that of a single cylinder because of the weakness and the inclination of the wake vortices. In regime 2 (\( 240 \leq Re \leq 300 \)), the RMS lift coefficient is increased significantly and it is greater than that on a single cylinder. The highest RMS lift coefficient occurs at the upper boundary Reynolds number in regime 2. The RMS lift coefficient is reduced in regime 3 (\( 320 \leq Re \leq 380 \)) because of the dislocation of the vortices and increased again in regime 4 because of the ceasing of the vortex dislocations in this regime.

![Figure 4-15](image_url) The change of phase angle between lift coefficients of two downstream cylinders 3 and 4.
Figure 4-16 Variation of the force coefficients with Reynolds numbers. (a) Mean $C_D$, (b) Root Mean Square $C_L$.

4.4.4 Strouhal number

Vortex shedding frequency is investigated by carrying out Fast Fourier Transform (FFT) analysis of the lift coefficients. It is well understood that the peak frequency in the lift force spectrum corresponds to the vortex shedding frequency (Lam et al., 2003b). Carmo and Meneghini (2006) found that the Strouhal numbers of two cylinders in tandem arrangement are the same when the space ratio is in 0D ~ 8D. In this study (with spacing ratio of 2), it is found that the Strouhal numbers of the four cylinders are within 2% over all the Re range considered. Figure 4-17 compares the FFT spectra of the force coefficients on cylinder 1 and cylinder 4 at Re=500 in regime 4, with those at Re=280 in regime 2. They are normalized by the respective maximum value on the spectra, so that the peak values correspond to 1. It is observed in Figure 4-18 that the peak frequencies of the drag force are identical to those of the lift force for both the upstream and the downstream cylinders. This is believed due to the synchronization in vortex shedding for cylinders in close proximity (Lam et al., 2003a, Carmo et al., 2008). Apart from the peak frequency being about 0.2, there is another very low peak frequency on each of the lift spectrum in Figure 4-17 (a) and (b). This very low secondary frequency is the beating frequency of the force. Similar feature can be seen in Figure 4-17 at Re=480, where both the lift coefficients increase and decrease every non-dimensional time of
about 50, corresponding to a frequency of 0.02, which is close to the FFT frequency of 0.021 observed in Figure 4-17 (a) and (b). When no beating happens at Re = 280, as shown in Figure 4-17, there is only one sharp peak in the FFT spectrum of either the drag or the lift force in Figure 4-17 (c).

The variations of the Strouhal number with Reynolds number for cylinders 1 and 4 are shown in Figure 4-18 together with the Strouhal number of a single cylinder. The Strouhal number values for individual cylinders are consistently smaller than that of a single cylinder obtained in the present study, except at Re=200–220 where the wake of the single cylinder transits from 2-D to 3-D. Similar to those in the force coefficients, the discontinuities in Strouhal number occur at the boundary Reynolds number between any two flow regimes. When the wake of the four-cylinder system transits from regimes 1 to 2 at Re=240 and regimes 2 to 3 at Re=320, the Strouhal number also decreases suddenly. The decrease in the vortex shedding frequency when the three dimensional structure appears agrees with the report by Carmo and Meneghini(2006) in the study of flow around two cylinders in tandem for spacing ratios between 3D and 5D.
4.5 Conclusions

Uniform flow past four circular cylinders in an in-line square configuration with a constant spacing ratio of 2 is studied numerically by directly solving the three-dimensional Navier-Stokes equations using an open source code of OpenFOAM®. The focus of this study is to examine the effects of Reynolds number on flow regimes and hydrodynamic forces on the cylinders. A range of Reynolds number from 100 to 500 is considered. The results of the study are summarized as follows:

1. The flow can be classified into four regimes based on Reynolds number. The flow in regime 1 (100 \( \leq Re \leq 220 \)) is characterized by the occurrence of the weak and inclined spanwise vortices. The streamwise vortices are also very weak. The flow in regime 2 (240 \( \leq Re \leq 300 \)) is characterized by the regular wavy spanwise vortices and rib-shaped streamwise vortices. The wavelength of the spanwise vortices is about 1.2. The wake flow in this regime is similar to the transition mode B of a single cylinder. The flow in regime 3 (320 \( \leq Re \leq 380 \)) is characterized by strong vortex dislocations in the wake of the cylinders and the flow regime 4 (400 \( \leq Re \leq 500 \)) is characterized by the ceasing of vortex dislocations and strong streamwise vortices. The flow between the upstream and the downstream cylinders is 2-D in regimes 1, 2 and 3 and becomes 3-D in regime 4.

2. In regime 1 (100 \( \leq Re \leq 220 \)) the four cylinders largely behave as a single structure, only one vortex street is formed behind the four cylinder array. This is because the shear layers at the inner sides of the two tandem cylinder pairs are
too weak to interact with the shear layers from outer sides and quickly merged
with the shear layers of the same signs from outer sides of the two tandem
cylinder pairs.

3. Different from a single cylinder, the wake flow behind the four-cylinder array
becomes 3-D at much smaller Re (even at Re=100) than the critical Re for wake
transition to 3-D behind a single cylinder. This is because vortex shedding
around the four cylinder array at a small spacing ratio shares similar
characteristics of vortex shedding around an equivalent single cylinder of a large
effective diameter at a larger effective Re (approximately 3 times).

4. In regime 2, two distinct vortex streets form behind the four cylinder array. This
is because the shear layer from the inner side of each tandem cylinder pair is
strong enough to interact with the shear layer from the outer side of the same
tandem cylinder pair, resulting in a Kármán vortex street behind each of the two
tandem cylinder pairs. The stable anti-phase synchronization of the two Kármán
vortex streets and the mode B type three-dimensional instability behind the rear
cylinders of the tandem pairs are believed to be responsible for the regular wavy
spanwise vortices and rib-shaped streamwise vortices in regime 2.

5. The transition from regime 1 to regime 2 behind the four cylinder array is
believed to be induced by onset of three-dimensionality behind the rear cylinder
in each tandem pair of cylinders in the array. The transition from regime 1 to
regime 2 behind the four cylinder array shares many common features with the
wake transition from mode A to mode B of a single cylinder, and with the
fundamental modes of the wake transition to turbulence for two tandem
cylinders.

6. Similar to regime 2, two Kármán vortex streets also exist in regime 3. The two
Kármán vortex streets in regime 3 are mostly synchronized in an in-phase mode,
intermittently switching to an anti-phase synchronization. The interaction
between the two Kármán vortex streets in regime 3 is clearly stronger than that
in regime 2. The strong interaction between the two Kármán vortex streets and
phase difference of vortex shedding at different locations along the spanwise
direction are the major causes for the severe vortex distortions observed in
regime 3.

7. There are four interacting wakes in regime 4. The strong interactions between
the four wakes lead to many fine-scale vortices in the wake behind the four cylinder array. The transition from regime 3 to regime 4 is induced by the onset of vortex shedding from two upstream cylinders in the two tandem cylinder pairs.

8. When the flow transits from one regime to another, significant changes occur in the force coefficients. The mean drag coefficients on the two downstream cylinders are much smaller than those on the two upstream cylinders because the former are in the wake of the upstream cylinders. The RMS lift coefficients of the two downstream cylinders are relatively small in regimes 1 and 3 because of the inclination and the weakness of vortices in regime 1 and the vortex dislocation in Regime 3. The RMS lift coefficients of the two downstream cylinders are relatively large in Regimes 2 and 4 because the spanwise vortices in these two flow regimes are parallel to the cylinders and there are no vortex dislocations. The peak frequencies of the lift force and the drag force are found to be almost the same for any single cylinder in the array. It is also found that the Strouhal number values for individual cylinder in the array are within 2% difference over the Reynolds number range considered, due to the synchronized vortex shedding from the cylinders.

Acknowledgments

This work was supported by Australian Research Council Discovery Grant (Project ID: DP110105171) and was also supported by iVEC through the use of advanced computing resources located at iVEC@Murdoch. F.T. would like to acknowledge the support of the Australian Government and The University of Western Australia by providing SIRF and UIS scholarships for a doctoral degree.

4.6 References


Chapter 5

Oscillatory flow regimes around four cylinders in a square arrangement under small KC and Re conditions

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Abstract: Sinusoidally oscillatory flow around four circular cylinders in an in-line square arrangement is numerically investigated at Keulegan-Carpenter number ranging between 1 and 12 and Reynolds number between 20 and 200. A set of flow patterns is observed and classified based on known oscillatory flow regimes around a single cylinder. These include six types of reflection symmetry regimes to the axis of flow oscillation, two types of spatio-temporal symmetry regimes and a series of symmetry breaking flow patterns. In general, at small gap distances, the four structures behave more like a single body and therefore, the flow fields are similar to those around a single cylinder with a large effective cylinder diameter. With the increase of gap distance, flow structures around each individual cylinder in the array start to influence the overall flow patterns and thus, the flow field shows a variety of symmetry and

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asymmetry patterns as a result of vortex and shear layer interactions. The characteristics of hydrodynamic forces on individual cylinders as well as on the cylinder group are also examined. It is found that the hydrodynamic forces respond similarly to the cylinder proximity and wake interferences as the flow field.

5.1 Introduction

5.1.1 Motivation and Objectives

Sinusoidally oscillatory flow around a circular cylinder has been studied extensively for decades due to its relevance to engineering applications, such as wave loads on cylindrical marine structures (Maull and Milliner, 1978, Williamson, 1985, Bearman et al., 1985, Obasaju et al., 1988, Sarpkaya, 2002, Saghafian et al., 2003) and rich physics displayed by the flow, especially at relatively low oscillation amplitude and frequency (Honji, 1981, Tatsuno and Bearman, 1990, Dütsch et al., 1998, Elston et al., 2006, An et al., 2011). In contrast, oscillatory flow around multiple cylinders has not attracted attention it deserved. Multiple cylinders are commonly arranged in groups due to functional requirements in many offshore engineering applications where the structures are routinely exposed to oscillatory flow induced by waves. For example, a tension leg platform or a semisubmersible platform often has four columns that are spaced about three or four diameters apart. Riser systems of floating offshore platforms often comprise of closely spaced multiple pipes. Four cylinders in a square arrangement are unique in terms of flow characteristics they display. This is because it includes a pair of tandem cylinders in a side-by-side arrangement, as well as two pairs of staggered cylinders, thus both wake and cylinder proximity interferences may present.

A steady approaching flow around multiple structures has attracted much research attentions because of its unique characteristics of fluid forces and flow features. For example, it has been observed that when two or more circular cylinders are arranged close to each other in a steady flow, the proximity interference and the wake interference lead to repulsive forces and biased vortex shedding (Zdravkovich, 1987, Hu and Zhou, 2008). Oscillatory flow around multiple cylinders is expected to exhibit similar responses to these interferences as steady flow. However, our understanding is limited by a scarcity of research on the subject. This motivates the present study of oscillatory flow around four cylinders in a square arrangement, as shown in Figure 5-1.
Oscillatory flow around a circular cylinder is mainly governed by two dimensionless parameters, namely the Keulegan-Carpenter number $KC$ and the Reynolds number $Re$. The $KC$ and $Re$ are conventionally defined as,

$$ KC = \frac{U_m T}{D} \quad \text{and} \quad Re = \frac{U_m D}{\nu}, $$

where $D$ is the diameter of the cylinder, $\nu$ is the kinematic viscosity of the fluid, $U_m$ and $T$ are the amplitude and period of the velocity oscillation respectively. The ratio of $Re$ and $KC$, known as the frequency parameter or Stokes number ($\beta = \frac{D^2}{\nu T}$) is also often referred to in literature. For oscillatory flow around a four-cylinder array, flow features are also affected by the gap ratio which is defined as the ratio of gap distance, $L$, to the cylinder diameter (refer to Figure 5-1),

$$ G = \frac{L}{D}, $$

It should be pointed out that the flow is also dependent on the angle between the direction of oscillatory flow and the cylinder-array arrangement. However this effect is not investigated in the present study. The main aim of the present work is to investigate the oscillatory flow features around four circular cylinders arranged in an in-line square arrangement, as shown in Figure 5-1. The study focuses on comparatively small $KC$ and $Re$ conditions which are bounded by $KC \in [1, 12]$ and $Re \in [20, 200]$, at gap ratio ranging from 0.5 to 4.

5.1.2 Previous studies

At low $KC$ and $Re$ values, an oscillatory flow around a single cylinder may present four kinds of two-dimensional (2-D) symmetries (Elston et al., 2004, Elston et al.,
2006): one reflection symmetry about the axis of oscillation, $K_x$, and two spatio-temporal symmetries, $H_1$ and $H_2$, along with a ‘basic state’, where all of this three symmetries are preserved. By introducing an oscillatory flow in the y-direction $U_y(t) = U_m \sin(2\pi t/T)$, the symmetry patterns identified for oscillatory flow around a single cylinder can be represented through vorticity component in the axial direction of the cylinders, $\omega_z$, as

$$K_x : \quad \omega_z(x, y, t) = \omega_z(-x, y, t), \quad (5-3)$$

$$H_1 : \quad \omega_z(x, y, t) = -\omega_z(x, -y, t + T/2), \quad (5-4)$$

$$H_2 : \quad \omega_z(x, y, t) = \omega_z(-x, -y, t + T/2). \quad (5-5)$$

![Figure 5-2](image)

Figure 5-2 The three symmetries $K_x, H_1, H_2$ of the $z$-vorticity component of the oscillatory flow (in y-direction) around a fixed circular cylinder at $KC=11$ and (a), Re=60 along with the coordinate system; (b), Re=80; and (c), Re=100; Instantaneous vorticity contours are obtained from 2-D DNS at 100th oscillation; Contours vary from $\Omega = -0.5$ (cold blue colors & dashed lines) to $\Omega = 0.5$ (warm red colors & dashed lines) with cutoff level between $\Omega=\pm0.1$.

where dimensionless $\omega_z$ is determined as the curl of velocity vector $\vec{U} = (U_x, U_y)$,

$$\omega_z = \nabla \times \vec{U} \left( \frac{D}{U_m} \right), \quad (5-6)$$

These symmetries are illustrated based on the present numerical results at $KC = 11$.

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and Re = 60, 80, and 100 respectively in Figure 5-2. It is noted that both $H_1$ and $H_2$ are preserved in the $K_x$ symmetry. It was also observed by Elston et al. (2006) that the $K_x$ symmetry in the cross-sectional flow fields around a single oscillatory cylinder is identical to the basic state with full symmetry.

Figure 5-3 Flow regimes classified by Tatsuno and Bearman (1990); Regime A: symmetric with vortex shedding, two-dimensional; $A^*$: symmetric and attached, two-dimensional; B: longitudinal vortices, three-dimensional streaked flow; C: rearrangement of large vortices, three-dimensional; D: transverse street, three-dimensional; E: transverse street with irregular switching, three-dimensional; F: diagonal double-pair, three-dimensional; G: circulatory flow streaming which intermittently changes directions, three-dimensional.

Comprehensive flow features induced by sinusoidal oscillations of a circular cylinder in an otherwise stationary fluid at low $KC$ and low $\beta$ were experimentally identified by Tatsuno and Bearman (1990). Eight flow regimes were classified within $1.6 \leq KC \leq 15$ and $5 \leq \beta \leq 160$. The flow regime map in the range of $KC \leq 12$ and $Re \leq 400$ is reproduced for the convenience of discussions in Figure 5-3 as a function of $KC$ and $Re$. Among the flow regimes, flows in regime A and $A^*$ are two-dimensional and symmetric to the direction of motion, with vortex shedding occurring in regime A but not in $A^*$. Regime B is featured with the so called ‘streaked flow’ along the axis of the cylinder, which is comprised of equally spaced streaks of mushroom-shape flow structures (Honji, 1981). Although regime B flows are three-dimensional, cross-sectional flows (in $x$-$y$ plane) preserve $K_x$ symmetry (Elston et al., 2006). Flows in regimes C ~ G are apparently three-dimensional and also break the two-dimensional $K_x$ symmetry. A key observation in regime C is that the vortex field does not synchronize with the oscillation period, but are rather rearranged into large vortices with a secondary period before emanating in the directions of motion. The transverse vortex streets are found in regime D and E, where vortices are obliquely convected to one side of the axis of oscillation.
Irregular switching of the convection direction is evolved in regime E. Diagonal double-pair vortices are featured in regime F. The $H_1$ symmetry is largely preserved while $K_x$ and $H_2$ symmetries are broken in regime C, D and E. The $H_2$ symmetry is roughly preserved, but $K_x$ and $H_1$ symmetries are broken in regime F. The flow field in regime G breaks all the $K_x$, $H_1$ and $H_2$ symmetric patterns and is chaotic and characterized by a circulatory flow streaming and irregular three-dimensional flow structures.

Many experimental and numerical studies have been carried out within the parameter ranges covered in the map of regimes shown in Figure 5-3. Some of these studies are summarized in Table 5-1. The flow regimes listed in Table 5-1 were guesstimated based on the ranges of $KC$ and $\beta$ where they were not provided in the original study. A significant amount of knowledge has been gained through these studies, which has been reviewed by Bearman (1984) and Elston et al. (2006) and thus is not repeated here. Only those numerical studies that are relevant to the present study are briefly reviewed here. It has been generally accepted that two-dimensional (2-D) numerical models are able to predict oscillatory flow features at a reasonably wide range of flow regimes, including the regimes that harbor three-dimensional (3-D) flows. By solving the 2-D Navier-Stokes (NS) equations, Justesen (1991) found that the simulated oscillatory flow structures around a cylinder were in good agreement with those observed in experimental flow visualizations at about $\beta \leq 250$ with small $KC$ numbers. The calculated drag and inertia coefficients for $\beta = 196$, 483 and 1035 and $0 < KC < 26$ were found to agree with the experimental data well. Lin et al. (1996) investigated oscillatory flow around a circular cylinder at a fixed $\beta = 76$ using a 2-D discrete vortex method. It was reported that all major vortex-shedding regimes that were observed in the experiments at $KC$ values up to 30 were well reproduced by the numerical simulation. Dütsch et al. (1998) reproduced regimes A, E and F by the 2-D simulations, and reported a good comparison of drag and inertia coefficients with experimental data at $\beta = 35$ (see Figure 5-3) and $KC \leq 15$. Nehari et al. (2004) compared the 2-D and 3-D numerical results in regimes D and F. It was found that the in-line force component (in the direction of oscillatory flow) is only weakly affected by the three-dimensional effect. It was also found that most of the cross-sectional vortex streets are induced by two-dimensional instabilities and can be reproduced by pure two-dimensional simulations. The numerical results obtained from 2-D numerical studies appear to suggest that three-dimensionality has limited effects on the principle cross-sectional flow features and the in-line force at relative low $KC$ and $\beta$ values.
### Reference Method \( KC \) and \( \beta \) Regime

<table>
<thead>
<tr>
<th>Reference</th>
<th>Method</th>
<th>( KC ) and ( \beta )</th>
<th>Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honji (1981)</td>
<td>Exp.</td>
<td>-</td>
<td>B</td>
</tr>
<tr>
<td>Hall (1984)</td>
<td>Analytical</td>
<td>-</td>
<td>B</td>
</tr>
<tr>
<td>Tatsuno and Bearman (1990)</td>
<td>Exp.</td>
<td>( 0 &lt; KC &lt; 15, \ 0 &lt; \beta &lt; 170 )</td>
<td>A*-F</td>
</tr>
<tr>
<td>Justesen (1991)</td>
<td>Num.</td>
<td>( 0 &lt; KC \leq 26, \ 196 \leq \beta \leq 1035 )</td>
<td>-</td>
</tr>
<tr>
<td>Lin et al. (1996)</td>
<td>Num.</td>
<td>( KC \leq 30, \ \beta = 76 )</td>
<td>A*,A,B,E,G</td>
</tr>
<tr>
<td>Iliadis and Anagnostopoulos (1998)</td>
<td>Num. (2-D)</td>
<td>( 0 &lt; KC &lt; 15, \ 6 &lt; \beta &lt; 100 )</td>
<td>A-F</td>
</tr>
<tr>
<td>Dütçh et al. (1998)</td>
<td>Exp. &amp; Num. (2-D)</td>
<td>( KC \leq 30, \ \beta = 20, \ 35 )</td>
<td>A, E, F</td>
</tr>
<tr>
<td>Uzunoğlu et al. (2001)</td>
<td>Num. (2-D)</td>
<td>( KC \leq 8, \ \beta = 35 )</td>
<td>A*,A,C,E,F</td>
</tr>
<tr>
<td>Nehari et al. (2004)</td>
<td>Num. (3-D)</td>
<td>( KC \leq 8.5, \ \beta = 20 )</td>
<td>D, F</td>
</tr>
<tr>
<td>Anagnostopoulos and Minear (2004)</td>
<td>Num. (2-D)</td>
<td>( 0.1 \leq KC \leq 6, \ \beta = 50 )</td>
<td>A*,A,E</td>
</tr>
<tr>
<td>Elston et al. (2006)</td>
<td>Num. (2-D&amp;3-D)</td>
<td>( KC \leq 10, \ \beta \leq 100 )</td>
<td>A*,A,B,C,D</td>
</tr>
<tr>
<td>Scandura et al. (2009)</td>
<td>Num. (3-D)</td>
<td>( KC = 10, \ \beta = 20&amp;50 )</td>
<td>F,G</td>
</tr>
</tbody>
</table>

Table 5-1 A brief summary of reviewed studies of oscillatory flow around a circular cylinder; Exp. = experimental; Num. = Numerical; the last column was guesstimated based on \( KC \) & \( \beta \) provided if not specifically mentioned in the original paper.

Studies on oscillatory flow around two or more cylinders have not been documented extensively in literature. Some available experimental and numerical investigations on multiple cylinders in oscillatory flow are summarized in Table 5-2. The alignment angle \( \alpha \) listed in Table 5-2 is the angle between the flow and cylinder array arrangement.

Williamson (1985) carried out an experimental study to investigate the synchronization of vortex shedding of two oscillatory cylinders in still fluid and measured the forces induced by the oscillatory flow on the two cylinders. Uzunoğlu et al. (2001) investigated the flow fields and force coefficients for two cylinders in side-by-side and tandem configurations. It is only until recently that oscillatory flow around two or four cylinders has attracted reasonable research interests, mainly based on 2-D numerical models. Chern et al. (2010) and Chern et al. (2013) simulated oscillatory flow past two side-by-side square cylinders and four circular cylinders in staggered and in-line arrangements, respectively. It was found that the gap flow between the cylinders has significant effects on the flow field and hydrodynamic forces on the cylinders. Yang et al. (2013) investigated oscillatory flow around a pair of cylinders of unequal diameters based a 2-D model. The influence of gap ratio and positional angle on the flow field and hydrodynamic forces were investigated. By solving the 2-D Navier-Stokes equations using a finite element method, Zhao and Cheng (2014) investigated oscillatory flow around a two-cylinder system in both side-by-side and tandem arrangements at two Reynolds numbers of 150 and 100. They identified several combined flow regimes based on the flow regimes observed around a single cylinder, as well as some new flow features, such as Gap Vortex Shedding (GVS), where the vortices only shed from the gap side of the system. So far, there has not been a systematic study on flow regimes around four cylinders, to the best knowledge of the authors.
In the present study, a 2-D numerical model is employed to investigate oscillatory flow regimes around a four-cylinder array in the parameter range of $KC \in [1, 12]$ and $Re \in [20, 200]$ with a $KC$ increment of 1 and Re increment of 20. Although relatively low Re values are chosen to avoid three dimensionality of the flow, three-dimensional flows are still expected within a narrow range of Re and $KC$ covered in this study. It is expected that the 2-D model is sufficient to reveal the occurrence of various flow regimes which are related to the two-dimensional instabilities (Nehari et al., 2004), and this is also confirmed by a brief comparison of the 2-D and 3-D numerical results around the four-cylinder array investigated in the present study. The remainder of the paper is organized in the following manner. In § 2, the governing equations, the numerical model and the model validations are presented, while § 3 presents the flow regime classifications. Drag and inertia coefficients are discussed in § 4, along with a brief discussion on wall vorticity. Finally, major conclusions are drawn in § 5.

### 5.2 Numerical method and model validation

#### 5.2.1 Numerical model

Oscillatory flow around the four-cylinder array shown in Figure 5-1 is simulated by solving the 2-D Navier-Stokes (NS) equations. The dimensionless form of 2-D NS equations for incompressible flow in the Cartesian coordinate system can be expressed as (An et al., 2011),

$$
\frac{1}{KC} \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_x}{\partial y} + \frac{\partial p}{\partial x} = \frac{1}{Re} \left( \frac{\partial^2 U_x}{\partial x^2} + \frac{\partial^2 U_x}{\partial y^2} \right) \tag{5-7}
$$

$$
\frac{1}{KC} \frac{\partial U_y}{\partial t} + U_x \frac{\partial U_y}{\partial x} + U_y \frac{\partial U_y}{\partial y} + \frac{\partial p}{\partial y} = \frac{1}{Re} \left( \frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_y}{\partial y^2} \right) \tag{5-8}
$$

$$
\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0 \tag{5-9}
$$
where $U_x$ and $U_y$ are the velocity components in the $x$- and $y$-directions, respectively, $t$ is time, and $p$ is kinematic pressure. The finite volume method is used and the pressure-velocity coupling is achieved following the Pressure Implicit with Splitting of Operators (PISO) method. The convection terms are discretised using the Gauss cubic scheme, while the Laplacian and pressure terms in the momentum equations are discretised using the Gauss linear scheme. Euler implicit scheme is adopted for the temporal discretisation. The NS equations are solved using the Open source Field Operation and Manipulation (OpenFOAM®) C++ libraries, which is a free source CFD package developed by OpenCFD Ltd.

A rectangular computational domain, as shown in Figure 5-4 (a), is employed in this study, with the cylinder array being placed at the center of the domain. The initial values for flow velocity and pressure in the whole domain are set to zero. Flow velocity and pressure boundary conditions on the bottom boundary are specified as

$$ U_y(t) = U_m \sin\left(\frac{2\pi}{T} t\right) $$

$$ U_x(t) = 0 $$

$$ \frac{\partial p}{\partial y} = U_m \frac{2\pi}{T} \cos\left(\frac{2\pi}{T} t\right) $$

At the top boundary, the velocity gradients in the $y$-direction and the pressure are set to zero. The symmetry boundary condition is applied at the two lateral boundaries that

Figure 5-4 Schematic representation of the computational domain and mesh distribution around a single cylinder.
are parallel to the flow directions and the no-slip boundary condition is adopted on the cylinder surfaces.

5.2.2 Mesh dependency check

The present numerical model is validated extensively against independent experimental and numerical results available in the literature for oscillatory flow around a single cylinder. First of all, suitable domain and mesh sizes are determined through a domain and mesh size dependency check. Then the numerical results of force coefficients, velocity distributions and flow regimes are compared with published experimental and numerical results.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Domain size</th>
<th>(N_v)</th>
<th>(N_c)</th>
<th>(\Delta/D)</th>
<th>(C_D)</th>
<th>(C_M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>32(D)×58(D)</td>
<td>41 948</td>
<td>120</td>
<td>0.003</td>
<td>2.11</td>
<td>2.43</td>
</tr>
<tr>
<td>Mesh 2</td>
<td>32(D)×58(D)</td>
<td>47 168</td>
<td>160</td>
<td>0.001</td>
<td>2.11</td>
<td>2.43</td>
</tr>
<tr>
<td>Mesh 3</td>
<td>100(D)×100(D)</td>
<td>176 064</td>
<td>140</td>
<td>0.001</td>
<td>2.11</td>
<td>2.42</td>
</tr>
<tr>
<td>Mesh 4</td>
<td>100(D)×100(D)</td>
<td>180 804</td>
<td>160</td>
<td>0.001</td>
<td>2.11</td>
<td>2.42</td>
</tr>
<tr>
<td>Mesh 5</td>
<td>100(D)×100(D)</td>
<td>186 264</td>
<td>180</td>
<td>0.00075</td>
<td>2.11</td>
<td>2.42</td>
</tr>
</tbody>
</table>

\begin{align*}
\text{Dütsch et al. (1998), 2-D numerical:} & \quad 98 304 \quad 384 \quad - \quad 2.09 \quad 2.45 \\
\text{Uzunoğlu et al. (2001), 2-D numerical:} & \quad 120 \times 120 \quad - \quad - \quad 2.10 \quad 2.45 \\
\text{Nehari et al. (2004), 2-D numerical:} & \quad 100 \times 36D \quad - \quad 48 \quad - \quad 2.10 \quad 2.43 \\
\text{Nehari et al. (2004), 3-D numerical:} & \quad 100 \times 36D \quad - \quad 48 \quad - \quad 2.13 \quad 2.47 \\
\text{Zhao and Cheng (2014), 2-D numerical:} & \quad 300 \times 30D \quad - \quad 84 \quad 0.0024 \quad 2.04 \quad 2.48 \\
\end{align*}

Table 5-3 Influence of the computational domain and mesh sizes on \(C_D\) and \(C_M\) of a single cylinder in oscillatory flow at \(\text{Re}, \text{KC} = (100, 5); N_v, \text{total mesh number}; N_c, \text{mesh number attached to the cylinder surface.}

The computational domain size and mesh dependency checks are carried out against oscillatory flow around a circular cylinder at \(\text{Re}, \text{KC} = (100, 5),\) where independent experimental and numerical results are available. Figure 5-4(b) illustrates a typical mesh distribution around the cylinder. Two domain sizes as detailed in Table 5-3 are considered in the mesh dependency check. Five meshes with cell numbers, ranging from 41 948 to 186 264 corresponding to domain size of 32\(D\)×58\(D\) and 100\(D\)×100\(D\), respectively, are generated by changing the mesh distribution around the cylinder surface, in both the radial and circumferential directions, and also in the far field. The calculated drag and inertia coefficients, \(C_D\) and \(C_M\) are compared with available data in Table 5-3. The drag and inertia coefficients are derived through the least square regression analysis of the inline force on the cylinder based on Morison equation (Morison et al., 1950),

\[
F_y = \frac{1}{2} \rho D C_D |U_y(t)| U_y(t) + \rho \frac{\pi D^3}{4} C_M \frac{dU_y(t)}{dt}
\]

(5-13)
where, $F_y$ is the force on the cylinder in the inline direction and is obtained by integrating the pressure and shear stress along the cylinder surface, and $\rho$ is the density of the fluid. It can be seen that despite the large differences in the domain and mesh sizes, the predicted drag and inertia coefficients are almost identical, suggesting that the numerical results are independent of the domain and mesh sizes employed here. Even Mesh 1 with the coarsest mesh on a smaller computational domain among the tested cases is sufficient for the flow condition of $(Re, KC) = (100, 5)$. The predicted drag and inertia coefficients showed less than 2% difference with those predicted by previous 2-D and 3-D numerical studies, confirming the weak three-dimensionality in the flow.

The mesh and domain size adequacy is also checked for the most extreme case of $(Re, KC) = (200, 12)$ covered in this study and the results are shown in Table 5-4. Again good convergence is observed.

Although all the meshes tested in Table 5-3 have been demonstrated to be sufficient for the flow condition considered, the relatively fine Mesh 4 with a large domain size is chosen in the simulations for the four-cylinder array. It is considered, first of all, that the large domain size of $100D \times 100D$ is very suitable for the planned simulations with the increased blockage ratio. Secondly the fine mesh in Mesh 4, with the number of cells more than 4 times of that in Mesh 1, is needed to resolve the fine flow structures in the gaps. The smallest mesh size in the radial direction of the cylinder (first layer next to the cylinder surface, $\Delta$) is $0.001D$, which is considerably smaller than that in Mesh 1 and the one employed in a relevant study by Zhao and Cheng (2014). The Mesh 4, which has a minimum non-dimensional distance of $y^+ = 0.03$ for the largest $Re = 200$ considered in this study ($y^+ = u_f \Delta / \nu$, where $u_f$ is the friction velocity), is considered to be sufficiently fine for all the cases considered in this study.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Domain size</th>
<th>$C_D$</th>
<th>$C_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>$32D \times 58D$</td>
<td>1.81</td>
<td>1.90</td>
</tr>
<tr>
<td>Mesh 5</td>
<td>$100D \times 100D$</td>
<td>1.80</td>
<td>1.88</td>
</tr>
</tbody>
</table>

Table 5-4 Mesh independent study at $Re = 200$, $KC = 12$

The predicted velocity distributions based on Mesh 4 are compared with the experimental data of Dütsch et al. (1998) for $(Re, KC) = (100, 5)$ in Figure 3-2. Since the tests reported by Dütsch et al. (1998) were carried out with an oscillatory cylinder in still water rather than an oscillatory flow around a fixed cylinder, the present data were converted into those in the coordinate system used in Dütsch et al. (1998) to facilitate the comparison, detailed in the caption of Figure 3-2. The calculated velocity profiles
along four straight lines of $\xi/D = -0.6, 0, 0.6$ and $1.2$ at two phase angles of $\psi = 180^\circ$ and $330^\circ$ are compared with the experimental data. The comparison suggests the numerical results agree well with measured data. The predicted velocity profiles are also compared with the numerical results reported by Zhao and Cheng (2014) and the predicted velocity profiles from the two numerical simulations match exactly.

![Graphical representation of velocity components](image)

Figure 5-5 A comparison between the present results (lines) of velocity components and the experimental data (symbols) by Dütsch et al. (1998) for $KC = 5, \beta = 20$. $\diamond/-\cdots$, $\xi/D = -0.6$; $\triangle/\cdots\cdots$, $\xi/D = 0.0$; $\circ/-\cdots\cdots$, $\xi/D = 0.6$; $\Delta/-\cdots\cdots$, $\xi/D = 1.2$. Since in Dütsch et al. (1998) study, the cylinder was forced to oscillate in otherwise standstill water, while in the present study the cylinder was kept still, the coordinates and velocity were transformed through, $\xi = y + A\sin(\psi)$; $\psi = \phi - 90^\circ$; $u_a = U_y(t) - U_m\sin(\phi)$; and $v_a = U_x(t)$; where $\xi$, $\psi$, $u_a$, and $v_a$ are the vertical coordinate, phase angle, velocity in vertical direction, velocity in horizontal direction, respectively; correspondingly, $y$, $\phi$, $U_y$ and $U_x$ are those parameters in the present study; $A$ is amplitude of the movement in experimental study, and $U_m$ maximum velocity in the present study.

The predicted drag and inertia coefficients at $\beta = 35$ (along $\beta = 35$ line shown in Figure 5-3) are compared with those reported by Kuehtz (1996) and Dütsch et al. (1998)
in Figure 5-6. Both drag and inertia coefficients derived from the numerical results are found to be in good agreement with the published data. The drag coefficient decreases monotonically with the increase of $KC$ before $KC = 4$, then it stays almost constant until $KC = 16$. On the other hand, $C_M$ does not show much change before $KC = 7$, followed by a sudden reduction until $KC = 16$. It is also observed that the numerical results agree with experimental data slightly better at low $KC$ $(\leq 6)$ than at high $KC$ values. This is believed to be due to the occurrence of 3-D flow at high $KC$ values. Nonetheless, the comparison is considered to be satisfactory.

![Figure 5-6](image_url) A comparison of the in-line force coefficients at $\beta = 35$; (a) drag coefficient; (b) inertia coefficient.

The oscillatory flow regimes identified by the present numerical model for a single cylinder are shown and compared with those reported by Tatsuno and Bearman (1990) in Figure 5-7. The simulated flow fields and force coefficients for a single cylinder are used as benchmarks for the simulations carried out for the four-cylinder array. The dashed lines shown in Figure 5-7 are the regime boundaries reproduced based on Tatsuno and Bearman (1990). Letter A in Figure 5-7 marks the location where the highest Re is detected for regime A at a constant $KC$, while all other letters indicate the location of the smallest Re detected for the corresponding regimes at the constant $KC$ values. It is seen that all the flow regimes are captured at the same corresponding areas identified by Tatsuno and Bearman (1990). Although the flows in regimes C, D, E and F are 3-D, they are successfully captured by the present 2-D model. This confirms the conclusion by Nehari et al. (2004) that the three dimensionality of the flow has a negligible effect on the cross-sectional flow features of oscillatory flow around a circular cylinder.

It should be noted, however, that the regime B flow, which is often observed in the range of Re $> 180$ and $2 < KC < 4.5$ and was firstly reported by Honji (1981), corresponds to the onset of three-dimensional instability. The cross-sectional flow fields
in regime B are similar to those in regime A* or A. Therefore, regime B cannot be distinguished from regime A* and A based on two-dimensional simulations. For this reason, Re is limited to approximately 200 in the present study.

Figure 5-7 Simulated flow patterns classified based on its regimes.

The predicted flow regimes are also compared with those reported by Tatsuno and Bearman (1990). Six selected cases representing the flow regime A*, A, C, D, E and F are visualized through streaklines in Figure 5-8. The streaklines are generated by releasing massless particles at 100 points around the cylinder surface with a frequency of eight times of the oscillatory flow frequency. Care is taken in generating the streaklines to make sure that the choice of the number of points around the cylinder and the frequency of releasing the particles would not affect the visualization of the flow fields. The massless fluid particles are released from the cylinder surface after the simulations become fully established, generally after $t = 90T$. The streaklines presented in Figure 5-8 agree qualitatively with the flow visualizations reported by Tatsuno and Bearman (1990). In regime A*, the fluid particles are transported away from the cylinder surface symmetrically along the axis of flow oscillation without vortex shedding. The flow patterns in regime A are similar to those observed in regime A* except that vortex shedding is detected. Regimes A* and A can be classified as the reflection symmetry, $K_c$. The rearrangement of large vortices in regime C and transverse vortex shedding (V-pattern) in regime D and E belong to the spatio-temporal symmetry $H_1$. It should be noted that regime C creates a quasi-periodic state with a well-defined second periodic mode (Elston et al., 2006). The flow changes between one pattern and its mirror image intermittently in regime E, while it is stable in regime D. The regime F falls into the $H_2$ symmetry pattern, with two branches of the vortex streets being
diagonally aligned in the opposite sides of the cylinder. Regimes $A^*$, $A$, $C$, $D$, $E$ and $F$ reported by Tatsuno and Bearman (1990) are all observed in the present simulations.

![Simulated flow patterns](image)

Figure 5-8 Simulated flow patterns represented by streaklines in different flow regimes for oscillatory flow past a single cylinder.

More model validations are presented in the Appendix and these tests demonstrate that the present numerical model is capable of predicting the force coefficients, velocity distribution and flow patterns of oscillatory flow past a single cylinder within $KC \in [1, 12]$ and $Re \in [20, 200]$. In the remaining part of this paper, we will simulate oscillatory flow around a four-cylinder array in a square arrangement. The primary aim of this study is to investigate the effect of cylinder proximity on the flow regimes around individual cylinders as well as around the cylinder array as a whole.
Figure 5-9 The classifications of flow regimes of oscillatory flow around four circular cylinders in an in-line square arrangement; (a), gap ratio \( G = 0.5 \); (b), \( G = 1 \); (c), \( G = 2 \); (d), \( G = 4 \); \( K_c \), \( H_1 \) and \( H_2 \) are the 2-D symmetries explained in section 5.1.2; \( N \) denotes that none of the symmetries were found, i.e., the state of symmetric breaking; \( \cdots \), boundary lines for basic state within \( K_c \) symmetry where both \( H_1 \) and \( H_2 \) are preserved; grey dashed lines are the boundary regimes found by Tatsuno and Bearman (1990) for a single cylinder.
5.3 Flow regimes around the cylinder array

A number of flow patterns are observed around a four-cylinder array for \(0.5 \leq G \leq 4\), \(20 \leq \text{Re} \leq 200\) and \(1 \leq KC \leq 12\). While a few of the observed flow patterns resemble, in many respects, the flow fields of a single cylinder, a number of new flow structures induced by the interactions of shear layers/vortices from individual cylinders are discovered and classified. Instead of creating new names for the flow features around the four-cylinder array, we prefer to classify the flow regimes based on established names of the flow regimes and symmetry patterns named by Tatsuno and Bearman (1990) and Elston et al. (2006).

![Figure 5-10](image)

Figure 5-10 An example of gap flow at \((G, \text{Re}, KC) = (0.5, 120, 8)\). Contours vary from \(U_y = -1.0\) (cold blue colors & dashed lines) to \(U_y = 1.0\) (warm red colors & dashed lines).

![Figure 5-11](image)

Figure 5-11 Measured velocity in the oscillating direction \(U_y / U_{in}\) at selected probes compared to the inlet velocity for case of \((G, \text{Re}, KC) = (0.5, 120, 8)\) at \(t = 98T - 99T\). (a) \(\cdot \cdot \cdot\), probe 1; \(\cdot \cdot\cdot\), probe 2; \(\cdot \cdot \cdot\), probe 3, \(\cdot \cdot\), inlet. (b), \(\cdot \cdot \cdot\), \(G=0.5\); \(\cdot \cdot\), \(G=1\); \(\cdot \cdot \cdot\), \(G=2\); \(\cdot \cdot\), \(G=\infty\).

The flow regimes for gap ratio of \(G = 0.5, 1, 2\) and 4 are mapped out on the Re-KC plane as shown in Figure 5-9, along with the symmetry patterns that describe the flow fields as a whole. The reflection symmetry and two spatio-temporal symmetries are labeled as \(K_x, H_1\) and \(H_2\), respectively, while the symmetry breaking is labeled as \(N\), i.e., none of the symmetry could be found. The areas at the bottom left corner enclosed by the dark dash-dotted lines (labeled as \(K_x\)) and the figure frame boundaries in Figure 5-9
belong to $K_x$ symmetry, while the areas beyond $N$ lines at the top-right corners of Figure 5-9(b), (c) and (d) belong to the symmetry breaking regime $N$. Regime $H_1$ flows are only observed at the smallest gap ratio of $G=0.5$ in an area enclosed by the $K_x$ line, $H_1$ line and the top and right figure frame. Regime $H_2$ flows are found at three larger gap ratios in the Re-$KC$ plane in Figure 5-9(b), (c) and (d), which fall into an area between $K_x$ and $N$ and are labeled as $H_2$ directly on the corresponding points. The boundaries for flow regimes of a single cylinder in oscillatory flow are also included as light dashed lines in Figure 5-9 for the purpose of comparison.

In the $K_x$ domain, the flow regimes around the cylinders are named based on the flow regimes identified by Tatsuno and Bearman (1990) for a single cylinder. For instance, regime 4A indicates four individual regime A flows with little influence from each other in the array. The structure of the flow in each regime will be explained in the following subsections.

### 5.3.1 $K_x$ symmetry

It is observed that the upper $K_x$ regime boundary lines for $G = 0.5$ and 1 are located at the top-right corners of Figure 5-9 (a&b) and are far different from the boundary lines of the flow regime A and D of the single cylinder. The reason for the dominance of regime $K_x$ at low gap ratios is mainly two folds. Firstly, oscillatory flow around the four-cylinder array at small gap ratios behaves in a similar way to oscillatory flow around a single cylinder with a large effective cylinder diameter. A large effective cylinder diameter corresponds to a small effective $KC$ number, which shifts the $K_x$ regime boundary line to the bottom-right and also suggests a more stable flow based on the flow regime chart (in the $\beta$-$KC$ plane) reported by Tatsuno and Bearman (1990).

Secondly, the jet-like flow through the cylinder gaps in the oscillatory flow direction plays an important role in maintaining the $K_x$ symmetry of the flow. Oscillatory flow structures around the cylinder array are examined to illustrate this point further. Figure 5-10 shows the contours of $U_y$ together with a few illustrative streamlines at eight instants with a time interval of $T/8$ in an oscillation period at $(G, \text{Re}, KC) = (0.5, 120, 8)$. A jet-like flow pattern is observed through the gap between two cylinder columns in all instants except for the two instants when the flow changes its directions, as seen from the upright streamlines.

Figure 5-11(a) shows the measured velocity at three locations along the oscillation
direction in the gap (probe 1 and 3 are in the middle point of two sideward cylinders, while probe 2 is at the origin of coordinates), together with the ambient flow velocity measured at the inlet of the computational domain. From the velocity contours and the measured velocity at the probe locations, it is observed that (1) the velocity magnitudes at the three probe locations are about twice the ambient velocity at the inlet of the computational domain; (2) there are phase differences between the ambient velocity and the velocities measured at the three probe locations; (3) the phase differences between the ambient flow and flow at probe 1 and 3 are somewhat larger than that between ambient flow and flow at probe 2, while the velocities at all three probe locations reach their peaks at almost the same time and (4) the duration of the flow acceleration phase at probe 2 is shorter than the duration of the deceleration phase at the same location. All these observations can be explained. The large velocity magnitudes at the three probe locations are clearly due to the blockage effects of the structure. The phase differences between the ambient velocity and the velocity measured at the three probe locations are the direct results of wall boundary layers around cylinder surfaces. The larger phase differences between the ambient flow and the flow at probe 1 and 3 locations than that between the ambient flow and the flow at probe 2 are because probes 1 and 3 are closer to the cylinder surface than probe 2. The closer to the wall boundary the larger the phase angle based on the Stokes boundary layer solution. The shorter duration of the acceleration phase at probe 2 is mainly induced by the combined effects of the phase difference between the velocity at the probe 2 and probe 1 (& 3) at small velocity magnitudes and the continuity constraint of the strong jet flow through the cylinder gaps at large velocity magnitudes that forces the velocity at the probe 2 to reach its peak value at approximately the same time as the velocities at probe 1 and probe 3 locations reach their peak values. The strong jet-like flow through the gap forms a distinct separation of the left two cylinders from their right counterparts and decreases the chance of vortex and/or shear layer interactions between the flows around left and the right cylinder columns. Therefore it plays a significant role in stabilizing the flow field in the \( K_x \) symmetry. We call this flow feature at low gap ratios the stabilization effect, since it provides stability to the flow in the streamwise (oscillation) direction.

The stabilization effect weakens with the increasing \( G \) and/or \( Re \), where vortices start to be shed alternatively from the inner sides of the cylinders.

Figure 5-11 (b) shows the variations of oscillation velocity in an oscillation period at
probe 1 for four gap ratios at \((\text{Re}, KC) = (120, 8)\). It is seen that both the differences in the magnitude and in the phase angle between the velocity at probe 1 and the ambient velocity at inlet gradually diminishes with the increase of the gap distance.

At \(G = 2\) and \(4\), on the other hand, the increase of gap ratio leads to the increase of individual flow behavior around the four cylinders, and more vortex interactions. As a result, much of the Re-\(KC\) plane at \(G = 2\) and \(4\) in Figure 5-9 is dominated by asymmetric flow fields of regime \(N\), because the symmetric pattern is simply not easy to achieve with independent vortex shedding from all of the four structures. This will be discussed in details later on.

Due to the interaction of flow fields from each of the four cylinders, six types of \(K_c\) symmetries are observed and are labeled as 2(A*-A*), 2(A-A), 4A*, 4A, 2(A-D) and 4D. Each of these symmetric patterns is explained below.

**Figure 5-12** An example of flow in regime 2(A*-A*) in \(K_c\) symmetry zone at \((G, \text{Re}, KC) = (1, 20, 5)\) observed in the oscillatory flow around four circular cylinders in an in-line square arrangement, illustrated by eight instantaneous vorticity contours at one oscillation period and by the massless particles advected from the surfaces of the cylinders. Contours vary from \(\Omega = -0.5\) (cold blue colors & dashed lines) to \(\Omega = 0.5\) (warm red colors & dashed lines) with cutoff level between \(\Omega = \pm 0.1\).

(a) \textit{Regimes 2(A*-A*) and 2(A-A)}

In regime 2(A*-A*) and 2(A-A), the left and right column of two cylinders behave like two elongated structures in a side-by-side configuration. The flow fields around both of the elongated structures fall into regime A* or A at small values of \(KC\) and \(\text{Re}\). Vorticity (\(\omega_z\)) contours and streaklines of a typical regime 2(A*-A*) flow with \((G, \text{Re},\)
$KC = (1, 20, 5)$ and a typical 2(A-A) flow with $(G, \text{Re}, KC) = (1, 80, 5)$ in one vortex shedding period are shown in Figure 5-12 and Figure 5-13, respectively. The released massless particles from different cylinders are denoted by different colors to visualize the mixing of particles as a consequence of flow interference. It is seen the shear layers from the two elongated side-by-side pairs are merged together for most of the time in one oscillation period, but are divided after flow reversal when the velocity of the fluid is relatively low, i.e., at $t = 99+1/8T$ and $+5/8T$. The vorticity strengths at the outer sides of the array are considerably smaller than those in the inner sides. The strong flow through the streamwise gap is an obvious feature of regime 2(A*-A*) and 2(A-A) flows. A similar feature of vortex shedding from the gap observed in Figure 5-13 was also reported in oscillatory flow around two side-by-side cylinders by Zhao and Cheng (2014) and was named as Gap Vortex Shedding (GVS) flow regime. Since weak vortex shedding is also observed from the outsides of the four-cylinder array, we named this regime 2(A-A).

Figure 5-13 An example of flow in regime 2(A-A) in $K_x$ symmetry zone at $(G, \text{Re}, KC) = (1, 80, 5)$.

The resemblance of the oscillatory flow features around a four-cylinder array at small gap ratios to those of a single cylinder is illustrated and explained with the aid of flow visualization shown in Figure 5-13. If we view the four-cylinder array as a single object represented by the shaded square shown at instant $100T$, pay attention to those large
merged vortices away from the cylinders and ignore the fine vortices close to the cylinder surfaces, then the flow field around the four-cylinder array is nothing but a regime A flow around a single cylinder, where vortex $A_1A_2$ and $B_1B_2$ are shed from each side of the square in an oscillation period. Therefore, as a whole, the flow in regime 2(A-A) around four cylinders in the square arrangement can be treated as regime A of the single cylinder case with a larger equivalent diameter. From the illustrations of streaklines in Figure 5-12 and Figure 5-13, on the other hand, regimes 2(A*-A*) and 2(A-A) are very similar to those of regime A* and A observed in Figure 5-8, despite the difference in the flow field close to the cylinder surfaces. The fine scale vortices near the cylinder surfaces are mostly generated through the gaps and are quickly merged with surrounding large vortices or simply dissipated, resembling flow features around a porous structure. At low and medium gap ratios, the flow fields around the four-cylinder array are characterized by the coexistence of influences from both individual cylinders and the cylinder array as a whole.

![Figure 5-14 An example of flow in regime 4A in K, symmetry zone at (G, Re, KC) = (2, 80, 5).](image)

Regime 2(A*-A*) and 2(A-A) dominate the Re-KC parameter plane shown in Figure 5-9 for gap ratios of 0.5 and 1, but this dominance weakens with the increase of the gap ratio and disappears at gap ratio of 4.

(b) **Regimes 4A* and 4A**

The 4A* and 4A regime occupy a large part of the Re-KC parameter plane at gap
ratio of 2 and 4 as shown in Figure 5-9 (c) & (d) respectively. Regime 4A* and 4A flows comprise of regime A* and A fluid flows around each of the four circular cylinders, respectively. A typical regime 4A flow with \((G, \text{Re}, KC) = (2, 80, 5)\) is visualized in Figure 5-14 through four snapshots in an oscillation period, together with the streakline flow pattern. In one oscillation period, a pair of counter-rotating vortices are generated at each side of an individual cylinder and this is distinctly different from regime 2(A-A) flows, where the vortex shedding in the transverse gaps is largely suppressed. The vortices shed in the transverse gaps in regime 4A tend to be convected outward sideways from the area enclosed by the cylinders, as seen from the streakline plot in Figure 5-14, where the fluid particles forms into a shape of a mushroom in each side of the transverse gap. Each mushroom shaped flow pattern is comprised of two circulation zones, as represented by two arrows in Figure 5-14. It is seen that each of these circulation zones corresponds to another circulation zone in the inner space surrounded by the four circular cylinders. These circulation zones are the direct results of vortex shedding and interaction of vortices shed in the transverse gaps.

\[
t = 99 + 1/4 \frac{T}{4} + 2/4 T
\]

Figure 5-15 An example of flow in regime 4A in \(K_x\) symmetry zone at \((G, \text{Re}, KC) = (2, 200, 5)\).

With the increase of \(KC\) and/or \(\text{Re}\), stronger vortices are found in both streamwise and transverse directions, and therefore the convections of fluid particle in these
directions are stronger, leading to increasingly complex streaming flow patterns. Figure 5-15 shows such an example at \((G, \text{Re}, KC) = (2, 200, 5)\), which has a higher value of Re than but identical values of \(G\) and \(KC\) to the flow case shown in Figure 5-14, and where stronger vortices and longer wakes are observed around each of the four cylinders. This suggests that vortex interactions between any two adjacent cylinders become more vigorous at Re = 200. The strong interactions between the wakes on the top two and bottom two cylinders, respectively, lead to four enlarged circulation zones which replace the mushroom-type flow in Figure 5-14. The limited gap space between cylinder 1 and cylinder 2 (also between cylinder 3 and cylinder 4) allows vortices to be shed from the gap sides but forces them to be convected horizontally outwards from the area enclosed by the four cylinders. It is interesting to observe that the convection of vortices behind the cylinder columns in the \(+y\)-direction is weaker than that in the \(-y\)-direction, resulting in a smaller wake area in the top than that in the bottom, which corresponds to the inclined transverse-gap flow towards \(+y\)-direction. This flow feature can be clearly observed from the streakline patterns shown in Figure 5-15. It is also observed that only a few fluid particles released from cylinder 2 and cylinder 3 are convected in the \(+y\)-direction. Most of these particles are trapped and circulated in the inner side circulation zones and are eventually convected outward sideways.

To investigate the transverse mass convection observed in Figure 5-15, the steady streaming velocity is obtained by averaging flow velocities over 10 consecutive oscillation periods, as given in Figure 5-16. The temporal mean velocity profile along \(x=4D\), as presented in Figure 5-16(a), demonstrated that the transverse mass convection observed in Figure 5-15 is indeed due to the large horizontal mass convection velocities directing away from the cylinder.

![Figure 5-16](image-url)
array. The magnitude of the mass convection velocity observed in the gap between cylinder 3 and 4 is as large as $0.4U_m$, as shown in Figure 5-16(b).

![Image of Figure 5-16(b)](image)

Figure 5-17 Time history of lift forces for two chosen cylinders for case at $(G, \text{Re}, KC) = (2, 200, 5)$.

Similar to its counterpart in regime A for the single cylinder, the convection of fluid particles in regime 4A does not change its direction once the asymmetry pattern about the $x$-axis has been developed. Figure 5-17 shows the time histories of the lift coefficient imposed by the flow fields shown in Figure 5-15, which is defined as $F_L = F_x/(0.5\rho U_m^2)$ with the $F_x$ being the force in the $x$-direction. Only those of the left pair of cylinders 1 and 2 are given because of the symmetric pattern. It is readily seen that the $F_L$ is quite stable over the time period of the simulation, suggesting a stable flow mode.

Regimes 4A* and 4A generally occur at smaller $KC$ values than those of regimes 2(A*-A*) or 2(A-A) at certain gap ratios. No regime 4A* or 4A is found at $G = 0.5$, and they seem to be a common feature at large gap ratios.

(c) **Regime 2(A-D)**

Regime 2(A-D) flows generally develop at relatively larger gap ratios with slightly higher $KC$ values than regime 4A as shown in Figure 5-9. Several cases of regime 2(A-D) flows are presented within the zone of $K_x$ symmetry. A flow in this regime, as the name suggests, has the characteristics of both regime A and regime D flows of a single cylinder. Figure 5-18 shows a typical regime 2(A-D) flow with $(G, \text{Re}, KC) = (4, 140, 6)$, where the flow patterns are presented by the vorticity contours in four equally spaced instants of time in an oscillation period and flow streaklines generated using the data in 10 oscillation periods.
Figure 5-18 An example of flow in regime 2(A-D) in $K_x$ symmetry at $(G, \text{Re}, KC) = (4, 140, 6)$.

The vortices shed from the inner sides of the cylinders 2 and 3 are considerably stronger than those shed from the outer sides in each half period. Consequently, the fluid mass is obliquely convected to the outer sides of oscillation and the flow field is classified as regime D. The flow fields around two bottom cylinders are similar to those around the two top ones observed in Figure 5-15 at regime 4A but with slightly more complex features. The fluid mass is not convected sideways through the horizontal gaps as shown in Figure 5-18, but instead two large circulation zones are formed in the streamwise gap. This is believed to be due to the strong interactions between the shed vortices as a result of increased gap space and the blockage of the inclined vortex streets from the top cylinders. The collisions of the streaklines from the top and bottom cylinders as indicated by the arrows S in Figure 5-18, result in three convectional paths of fluid mass indicated by arrows $S_1$, $S_2$ and $S_3$. The movement of fluid mass accompanying the vortices generated from the two bottom cylinders is complex but is highly influenced by the jet flow in the streamwise gap and thus falls into flow regime A. Since the flow field around the right hand two cylinders is actually a reflection of what occurs on the left, this regime as a whole is named 2(A-D). With several circulation zones, regime 2(A-D) presents a perfect $K_x$ symmetry.
(a) Time history of lift force at regime 2(A-D) for case at \((G, \text{Re}, KC) = (4, 140, 6)\)

(b) Flow fields of difference zones listed in (a)

(c) Time history of lift force at regime 2(A-D) for case of \((G, \text{Re}, KC) = (1, 140, 5)\)

Figure 5-19 Time history of lift forces and flow fields for regime 2(A-D).

Flow structures observed in Figure 5-18 are unstable as demonstrated by the lift coefficient along with the flow fields shown in Figure 5-19. The time histories of the lift coefficient on cylinder 1&2 exhibit five distinct time zones (A~E) and each of the time zones corresponds to an evolving stage of flow field as shown in Figure 5-19(b). The flow field around each of the four cylinders initially appears to be in regime D over a long time period zone A. In time zones B and C, the vortices shed from the two-bottom cylinders gradually move closer to the cylinders and the flow develops into a regime 2(A-D) flow. Meanwhile, the direction of the mean lift and the magnitude of the lift change several times, but the flow field continues to be \(K_x\) symmetry. However, regime 2(A-D) can be stable as given in Figure 5-19(c), where the time histories of the lift force
for cylinders 1 and 2 at \((G, \text{Re}, KC) = (1, 140, 5)\) remain regular and repeatable over the entire simulation period, indicating that the flow pattern remains stable \(K_x\) symmetry. An obvious beating characteristic of lift is found in the time history for small gap ratios (Figure 5-19, c), but the beating characteristics at large gap ratios is not as significant (Figure 5-19, a). This beating feature is due to the impingement of vortices shed from its neighboring cylinder in the same column and is one of the common features for cylinders with small gap ratios.

Regime 2(A-D) flows only occur at several specific values of \((G, \text{Re}, KC)\) and it can be deemed as a transitional regime from regime 4A to regime 4D.

Regime 4D flows generally develop at relatively larger gap ratios with slightly higher \(KC\) values than those of regime 2(A-D) flows as shown in Figure 5-9. The regime D flow around a single cylinder shows \(H_1\) symmetry. However, the regime 4D around the four-cylinder array, where a regime D flow occurs around each of the four cylinders is found in \(K_x\) symmetry.

One of the prominent features of a regime 4D flow is that vortices shed from each cylinder clearly deviate from the direction of flow oscillation, and tend to be convected transversely. Figure 5-20 shows the flow patterns at various stages in an oscillation period in regime 4D at \((G, \text{Re}, KC) = (2, 120, 7)\), together with the streaklines pattern of the flow. Due to the stronger vortices developed on the inner sides of cylinders than on the outsides and also due to the weakening of the stabilization effect, the shed vortices...
from each cylinder tend to move diagonally. As indicated by the elliptic circles at \( t = +2/4T \) and \( +3/4T \), when the flow oscillates upwards, three vortices are developed around each cylinder, with a pair of counter-rotating vortices along the inner sides and a single vortex along the outer sides of the cylinders. However, when the flow reverses, only a pair of vortices is shed along the inner sides of the cylinders. Hence, the flow fields are asymmetric to the \( x \)-axis and the fluid mass is convected downwards. Vortices around cylinder 1 and 2 (also around 3 and 4) appear to be similar, but because the movement of vortices in the transverse gaps is constricted by the space, the fluid mass convected from the top two cylinders is forced to circulate around the bottom two cylinders and then convected obliquely by the vortices developed from the two bottom cylinders.

**Figure 5-21** An example of flow in regime 4D in \( K_c \) symmetry zone at \((G, \text{Re}, KC) = (2, 160, 6)\).

Figure 5-21 shows the features of regime 4D flows when the flow fields are symmetric to the \( x \)-axis at \((G, \text{Re}, KC) = (2, 160, 7)\). It is seen that vigorous vortex pairing and merging take place in this case. For the convenience of discussions, at \( t = 99 + 1/4\) \( T \), three vortices surrounding cylinder 1 are named as \( A_1^o, B_1^o \) and \( C_1^o \), where ‘\( o \)’ denotes for the vertices generated from the last oscillation period. With the increase of velocity in the +\( y \)-direction, a new positive vortex named as \( N_1^+ \) is generated from the surface of cylinder 1. Vortex \( N_1^+ \) breaks \( B_1^o \) into two parts, i.e., \( B_{1-p1}^o \) and \( B_{1-p2}^o \), as \( B_1^o \) moves upwards along with the flow. Vortex \( N_1^+ \) starts to merge with \( A_1^o \) and \( C_1^o \) at \(+2/4\) \( T \) and a long positive vortex that wraps around the inner side of cylinder 1 is formed before \(+3/4\) \( T \). At the same time, a new negative vortex, named as \( N_1^- \), starts to emerge
from the cylinder surface. The $N_{1}^{-1}$ gradually grows in strength and cuts the positive vortex generated at $+3/4 \, T$ into two new vortices, $A_{1}^{0}$ and $C_{1}^{0}$, which replace the vortex $A_{1}^{0}$ and $C_{1}^{0}$. Meanwhile, $N_{1}^{-1}$, $B_{1-p1}^{o}$ and $B_{1-p2}^{o}$ begin to combine together and eventually, generate vortex $B_{1}^{o}$ at the start of the next period of oscillation. It is also observed at $+4/4 \, T$ that part of $B_{1-p2}^{o}$ is merged upwards into the sideway vortex generated from cylinder 2; while vortex $C_{1}^{o}$ also attracts vortices from cylinder 2 at $+2/4 \, T$. Except for the interaction described above, the individual regime D flow around each of the cylinders in the array does not interact with each other actively, and much of the merging and paring of vortices occurs among vortices from the same cylinder. Even when no vortex merging takes place, vortex interactions in the confined space between the transverse gaps are quite visible.

Figure 5-22 Steady streaming velocity averaged from flow fields in 10 oscillation periods at $(G, \, Re, \, KC) = (2, \, 160, \, 6)$; (a), velocity profile along $x=4D$; (b), velocity contour with streamlines.

One difference between regime 4D flows shown in Figure 5-20 and Figure 5-21 is that the flow fields around each of the cylinders are the same in Figure 5-21 while it is not the case for the regime 4D flow shown in Figure 17. Both types of regime 4D flows as shown in Figure 5-20 and Figure 5-21 are stable, as evidenced from the time history of lift (not shown here), mainly due to a mechanism that is similar to the stable sideway convection of fluid particles in regime D for a single cylinder.

The steady streaming velocity fields for the case shown in Figure 5-21 are given in Figure 5-22, where Figure 5-22 (a) shows the streaming velocity profile along $x=4D$ and Figure 5-22 (b) gives the contour of streaming $\bar{U}_{x}/U_{m}$ along with the streamlines. The cylinder array seems to attract fluid mass from an even larger area compared to that shown Figure 5-16, since all streamlines above the top two cylinders and below the bottom two cylinders are attracted towards the cylinder array and are directed away.
Regime 4D is only found at large space ratios (G=2 and 4 in the present study) and occupies an area in the Re-KC plane where regime D would normally occur for a single cylinder case. It involves vigorous vortex interactions, which result in complex flow features.

(e) Comparisons of the \( K_x \) symmetries

One of the interesting features for oscillatory flow around the four-cylinder array is that both \( H_1 \) and \( H_2 \) are frequently broken within \( K_x \) zones near the boundary lines as shown in Figure 5-9, for example, those cases in regimes 2(A-D), 4D, and even in regime 4A. This is different from the \( K_x \) symmetry for a single cylinder, where both \( H_1 \) and \( H_2 \) symmetry are satisfied within the \( K_x \) regime.

For a single cylinder, it has been demonstrated that as soon as the two-dimensional \( K_x \) symmetry is broken, three-dimensional secondary instabilities will develop, such as flow changing from regime A to regime D (Nehari et al., 2004, Elston et al., 2006). This leads to an interesting scenario that 2-D simulations can be used to predict the boundary line in the Re-KC plane where three-dimensional instabilities occur. In the four-cylinder case considered in the present study, it is expected that the onset of three-dimensional instabilities will occur within the zone of \( K_x \) symmetry rather than on the boundary, especially at large gap ratios, because the \( K_x \) symmetry around the four-cylinder array is not a basic state as that in the single cylinder case where both \( H_1 \) and \( H_2 \) are satisfied.

For this reason, a boundary line (dotted) is interpolated at each gap ratio in Figure 5-9 to distinguish the zone of the basic state of \( K_x \) from the rest of \( K_x \) symmetry. At \( G=0.5 \), the basic state boundary overlaps with the boundary line of \( K_x \). At \( G=1 \), \( H_1 \) and \( H_2 \) symmetries are only broken at two points of \((Re, KC) = (200, 4)\) and \((Re, KC) = (140, 5)\) where \( K_x \) symmetry is still observed. At those two small gap ratios, large parts of \( K_x \) symmetry in Figure 5-9 are similar to that of a single cylinder where both \( H_1 \) and \( H_2 \) are satisfied within the \( K_x \) symmetry regime. This is because the flow around cylinder array behaves similarly to that around a single cylinder as discussed earlier, and fine flow details around each cylinder has limited effect on the overall behavior of the flow.

In contrast, the boundary lines for the basic state at two large gap ratios deviate noticeably from the \( K_x \) boundary lines and are close to the boundary lines of regime A.
of the single cylinder case. The flow regime 2(A-D) and 4D for the four-cylinder array generally fall between the boundaries of the basic state and \( K_x \) symmetry regime and thus into an area where regime D would normally occur for a single cylinder. At those gap ratios, flow features around individual cylinders become important and the \( H_2 \) symmetry is broken beyond the basic state lines within the \( K_x \) regime simply because the \( H_2 \) symmetry for a single cylinder is broken in regime D. The \( K_x \) symmetry occurs beyond basic state lines because the interaction between the flows around the two cylinder columns is normally weak, inducing a flow field around one cylinder column being the mirror image of the other.

\[ t = 99 + 1/8 T + 2/8 T + 3/8 T + 4/8 T \]

Figure 5-23 \( H_1 \) symmetric flow for four circular cylinders in oscillatory flow at \((G, \text{Re}, K C) = (0.5, 200, 9)\).

5.3.2 \( H_1 \) symmetry (regime D)

The \( H_1 \) symmetry flow for the four-cylinder array is observed at low gap ratios and relatively high \( K C \) and \( \text{Re} \) as seen from Figure 5-23 at \((G, \text{Re}, K C) = (0.5, 200, 9)\). Figure 5-23 is deliberately arranged in a way so that the flow features in the top row are
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$H_1$ symmetric to those in the bottom row. It is seen that the vortices are shed from both the streamwise gaps and from the outside of the cylinder array every half of an oscillation period. The generation of the vortex named as $A^o$ at $t = 99+1/8 \ T$ is chosen to show how vortices are shed, interact, and pair in this regime. At $2/8 \ T$, three new vortices named $A_{p1}^n$, $A_{p2}^n$ and $A_{p3}^n$ are shed from the gap and outer sides of the cylinder array. The strengths of those vortices are comparable (with those from the gap being slightly larger), unlike those in $K_x$ symmetry. The small vortices of the same sign shed from the gap and from the outer sides of the cylinders are quickly merged together before being convected further from the cylinder array. The convection process of these vortices can be observed from the flow fields at $3/8 \ T \sim 6/8 \ T$. Vortex $A_{p1}^n$, $A_{p2}^n$ and $A_{p3}^n$ are fully merged at $7/8 \ T$ and a new large vortex named $A^n$ is formed at $8/8 \ T$. The generation of other large vortex cores is similar to the process just described. The large vortices generated are convected obliquely to one side of the axis of oscillation, in the same way as those observed in regime D in Figure 5-8. The streaklines on the right hand side of Figure 5-23, show similarities to those in regime D of a single cylinder. Therefore the resultant flow field falls into the $H_1$ symmetry and this kind of $H_1$ symmetry flow around the four cylinders is named as regime D, as given in Figure 5-9(a). It is easily seen that at $G = 0.5$, $H_1$ symmetry occurs because the four cylinders behave similarly to a porous single structure, with fine scale vortices being shed from different parts of this porous structure, but those of the same sign quickly merged to a single core. The vortices shed from the gap are significant but do not dominate the flow field. So the fluid mass is obliquely convected along with the merged cores.

Similar to regime D for a single cylinder, the convection of fluid particles in $H_1$ symmetry for the four-cylinder array is consistently inclined to one direction. We extended the simulation at $(G, \ Re, \ KC) = (0.5, 200, 5)$ for up to 280 cycles, the inclination direction has been maintained in the entire period of simulation.

It is worth to pointing out that $H_1$ symmetry for the four-cylinder array only occurs at the lowest gap ratio in this work. It is also observed from Figure 5-9 that the flow regime transits directly from regime $K_x$ to regime $N$ or regime $H_2$ at gap ratios of 1, 2 and 4, without $H_1$ symmetry flow. The reason is that in the four-cylinder array, the flow structures around the left and right columns of cylinders are almost identical due to the existence of a jet-like flow in the streamwise gaps and thus, flow around the two columns of the cylinders is equivalent to flow around two elongated side-by-side
structures. The $H_1$ symmetry can be observed around individual cylinders in regimes 2(A-D) and 4D, but as a whole, these regimes represent a $K_x$ symmetry.

$$t = 99 + 1/4 \, T + 2/4 \, T + 3/4 \, T + 4/4 \, T$$

Figure 5-24 $H_2$ symmetry for four circular cylinders in oscillatory flow resulting from vigorous boundary layer and vortex interactions at $(G, \text{Re}, KC) = (1, 180, 5)$.

5.3.3 $H_2$ symmetry

The $H_2$ symmetry flow occurs in a narrow area of the Re-$KC$ plane for $G = 1, 2$ and 4 as shown in Figure 9. There are two types of $H_2$ symmetry flow for the four-cylinder array investigated in this paper. The first one is due to vigorous boundary layer and vortex interactions at small gap ratios and the other is featured with gentle wake interactions.

Figure 5-24 shows consecutive stages of flow development over an oscillation period for the first type of $H_2$ symmetry at $(G, \text{Re}, KC) = (1, 180, 5)$, along with the streakline flow patterns. The process of vortex development and shedding can be clearly observed from the four snapshots. The snapshots with a half period difference are in the $H_2$ symmetry. Unlike the cases in $K_x$ symmetry, the vortices in the streamwise gaps start to be shed unevenly from, for instance, the inner sides of cylinders 2 and 3. So the flow fields close to the cylinders are quite chaotic with many fine scale vortices being shed, combined and dissipated. However, overall these fine scale vortices are evenly arranged in a very regular pattern in the $H_2$ symmetry. It is interesting to see that two large vortices of the same sign are developed in the far field shown in Figure 5-24.

Figure 5-25 shows another example of the first type of $H_2$ symmetry resulting from vigorous vortex interactions at $(G, \text{Re}, KC) = (2, 100, 10)$. The shear layer interaction in
this case is not as significant as that shown in Figure 5-24, and vortex merging occurs comparatively far from the cylinder surfaces. The merged vortex cores are convected in the oscillation direction and are roughly in $H_2$ symmetry, but the vortex cores in the $+\gamma$-direction appear to be slightly larger and closer to each other than those in the $-\gamma$-direction. The merged vortices are staggered at each side in a way resembling the Kármán vortex street. The flow fields in Figure 5-25 show a quasi-periodic state when they are convected away from the cylinder surfaces, with a secondary frequency that is clearly different from the velocity oscillation frequency. This is similar to the regime $C$ of a single cylinder case that was experimentally observed by Tatsuno and Bearman (1990) and numerically reproduced by Elston et al. (2006). One primary difference is that regime $C$ is in $H_1$ symmetry, but here the flow field is in the $H_2$ symmetry.

\[
t = 99 + \frac{1}{8} T + 2 T + \frac{3}{8} T + 4 T + \frac{5}{8} T + 6 T + \frac{7}{8} T + 8 T
\]

Figure 5-25 $H_2$ symmetry for four circular cylinders in oscillatory flow resulting from vigorous vortex interactions at $(G, Re, KC) = (2, 100, 10)$.

Figure 5-26 illustrates the second type of $H_2$ symmetry for four circular cylinders at $(G, Re, KC) = (4, 120, 8)$, where the shear layer and vortex interactions are less intensive than those shown in Figure 5-24 and Figure 5-25, as observed from the mixture of particles from different cylinders as they are convected away. It is seen that the convection of vortices generated around cylinder 2 and 4 is relatively constrained while those from cylinders 1 and 3 move further away from the cylinder surfaces. The vorticity contours illustrate that the merging of vortices from different cylinders only
occurs in the transverse gaps. Similar to those observed in Figure 5-24, the two large vortex cores away from the cylinder surface tend to rotate in the same direction. The fluid mass around cylinder 4 is partially forced to circulate around the cylinder itself and is partially convected upwards before it is attracted by the vortex developed on cylinder 3 and then convected upwards along with the vortex shed from cylinder 3. Similarly, fluid mass from cylinder 2 is transported downwards in the same way. Similar movements of mass and vortex from cylinder 2 and 4 can also be found in regime 4D.

The \( H_2 \) symmetry involves flow field interactions among the four cylinders and occurs only at a few cases near the symmetry breaking boundary lines. The flow features in the \( H_2 \) symmetry are quite spectacular due to the wake and proximity interferences in the cylinder array. It is noted that none of the \( H_2 \) flows, except two cases at \( G=1 \), is strictly in \( H_2 \) symmetry, especially close to the cylinder surfaces where vortex interaction is strong. Unlike those in the single cylinder case, the \( H_2 \) symmetry in the four-cylinder case is weakly stable.

\[
t = 99+1/4 \ T + 2/4 \ T + 3/4 \ T + 4/4 \ T
\]

Figure 5-26 \( H_2 \) symmetry for four circular cylinders in oscillatory flow resulting from gentle vortices interactions at \((G, \text{Re}, KC) = (4, 120, 8)\).

5.3.4 Symmetry breaking

In a large area at the top-right corner of the Re-\( KC \) plane with gap ratios of 1, 2 and 4 in Figure 5-9, none of the symmetry patterns discussed above are observed. The flow falling into this area is classified into regime N. The cross-sectional flow fields in regime N tend to be chaotic and correspond to those in regime G of a single cylinder. However the flow mechanisms responsible for those flow features are different. The
regime G of a single circular cylinder is generated by strong three-dimensionality in the spanwise direction at larger Re values than those of regime N flows for the four-cylinder array, while regime N flows are mainly attributed to the results of the interactions among the cross-sectional individual shear layers and vortices shed from each cylinder in the array and also to the weak three-dimensionality of the wake flow around the cylinder array, as further discussed in the next section.

\[(a), (G, \text{Re}, K_C) = (0.5, 200, 11)\]
\[(b), (G, \text{Re}, K_C) = (2, 200, 6)\]
\[(c), (G, \text{Re}, K_C) = (2, 180, 7)\]
\[(d), (G, \text{Re}, K_C) = (4, 180, 9)\]

Figure 5-27 Symmetric breaking for flow around four circular cylinders in oscillating flow and regime N; (a) symmetric breaking from regime D \((H_1\) symmetry); (b) from regime 4D, \(K_x\) symmetry; (c) from regime 4D, \(K_y\) symmetry; (d), regime N; pictures are not to scale and the dashed squares are 2D in size.

Much of the flow field beyond the line of \(N\) shown in Figure 5-9 is chaotic and thus cannot be classified easily. However, for the cases where \(K_C\) and Re are close or on the regime \(N\) lines, the flow is usually in a transitional breaking pattern from symmetry to asymmetry. Several interesting symmetry breaking flows are shown in Figure 5-27 by streaklines and the instant vorticity contours. Figure 5-27(a) shows a symmetry breaking case from regime D at \((G, \text{Re}, K_C) = (0.5, 200, 11)\) where streaklines and the wake in the \(-y\)-axis direction hooks more towards the \(x\)-axis than those in the \(+y\)-direction, leading to an asymmetric pattern. The symmetry breakings from regime 4A and 4D are shown in Figure 5-27(b) & (c). The streakline pattern in the \(-y\)-direction
shown in Figure 5-27(b) resembles the shape of a gold fish’s tail. From the comparison between Figure 5-27(c) and Figure 5-21, it is seen that the transverse convection of fluid mass by vortices generated from the right hand two cylinders are broken, while flow field around the two left cylinders are kept. A developed chaotic flow field is shown in Figure 5-27(d), where no symmetric pattern could be observed around any individual cylinder and the cylinder array as a whole.

The symmetry breaking for a single cylinder, which leads to regime G, can only evolve from $H_1$ and $H_2$ symmetries. In contrast, the symmetry breaking in the four-cylinder array (regime N) can evolve directly from $K$, symmetry.

5.3.5 3-D effects on onset of symmetry breaking

It has been demonstrated in the steady current that the existence of an additional cylinder in close proximity has a tendency to suppress or enhance the three dimensionality of the flow, depending on the distance of the cylinders (Papaioannou et al., 2006, Carmo et al., 2010). Although the range of $KC$ and Re was carefully selected to avoid 3-D flow regimes, 3-D effects are likely to occur within the parameter space covered in this study, especially in areas outside and just within the symmetric regimes. To investigate the possible effect of the three-dimensionality on the onset of symmetry breaking regime N, a limited number of 3-D simulations close to the symmetry breaking boundaries are carried out, as listed in Table 5-5, along with the reasons for the chosen cases. The simulations are based on the coarsest mesh 1 in Table 5-3, where a spanwise length of $10D$ is used, with 100 layers of the 2-D mesh.

<table>
<thead>
<tr>
<th>Case</th>
<th>G</th>
<th>Re</th>
<th>$KC$</th>
<th>Reason for the simulation</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.5</td>
<td>200</td>
<td>12</td>
<td>Symmetry breaking case &amp; most extreme case in the present study</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>180</td>
<td>12</td>
<td>Regime D, approaching symmetry breaking in terms of Re</td>
</tr>
<tr>
<td>III</td>
<td>0.5</td>
<td>200</td>
<td>9</td>
<td>Regime D, approaching symmetry breaking in terms of $KC$</td>
</tr>
<tr>
<td>IV</td>
<td>2</td>
<td>200</td>
<td>5</td>
<td>Regime 4A, vigorous vortex interactions &amp; close to symmetry breaking</td>
</tr>
</tbody>
</table>

Table 5-5 3-D simulations for selected cases

Three-dimensionality is observed for cases I and III, while it is hardly visible for cases II and IV. The iso-surfaces of spanwise vorticity ($\omega_z$) at an instant are shown in Figure 5-28. Obvious three-dimensionality is observed in Case I and Case III flows. The cross-sectional flow fields at a few selected cross sections are compared with the corresponding 2-D simulation results in Figure 5-29 for cases I and III. A number of observations are made from the limited number of 3-D simulations. First of all, the three-dimensionality does not appear to affect the onset of symmetry breaking regime N
based on the 3-D simulation results shown for case III and case IV. Both case III and case IV conditions are very close to the symmetry breaking boundaries shown in

![Flow features revealed by iso-surface of $\omega_z = \pm 0.5$ (represented by red and blue colours, respectively) from 3-D simulation.](image)

![Comparison on the cross-sectional flow fields between 3-D and 2-D simulations.](image)

Figure 5-28 Flow features revealed by iso-surface of $\omega_z = \pm 0.5$ (represented by red and blue colours, respectively) from 3-D simulation.

Figure 5-29 Comparison on the cross-sectional flow fields between 3-D and 2-D simulations.

Figure 5-9. It is seen that three-dimensionality develops for case III flow while case IV flow appears to be two-dimensional. The $K_{r}$ symmetry is clearly observed from the 3-D results for case IV and the cross-sectional flow fields agrees very well with those in Figure 5-15 by 2-D simulation. In spite of the strong three-dimensionality, the cross-
sectional flows of case III flow generally remain in the $H_1$ symmetry and can still be classified as regime D flows. It is also observed that 2-D simulation reasonably reproduces the main cross-sectional flow features. The three-dimensionality of case III flow is also observed by the direction change of transverse wakes along the spanwise direction of the cylinder array. Secondly the symmetry breaking case I flow shows the strongest three-dimensionality among the four 3-D simulation cases and is still classified as symmetry breaking flow based on 3-D results. Significant differences in wake flow structure exist between cross-sectional flows themselves and also between cross-sectional flows and the flow from 2-D simulation; however, some major flow structures are less influenced by the 3-D instability and some similarities can still be observed between 2-D and 3-D simulations. Thirdly, the flows outside the symmetry breaking regime (Case II, III and IV) show relatively weak three-dimensionality and can be classified based on 2-D simulation results.

5.4 Quantifications on the flow fields

5.4.1 Vorticity on the wall

As discussed from the above analysis, the oscillatory flow field around the four-cylinder array displays great diversity and complexity, which makes meaningful quantitative analyses more difficult. Nonetheless, the non-dimensional vorticity on the cylinder surface of selected cylinders in an oscillation period at $(Re, KC) = (160, 5)$ of the cylinder-array cases are compared with that of a single cylinder case in Figure 5-30. One outstanding observation is that the maximum vorticity on the inner side surface are consistently larger than those on the outer side surface for the cylinder array, and the difference between the vorticity on the inner and outer side surfaces appears to be dependent on the gap ratio. Another interesting feature is that when the flow around individual cylinders is in the same regimes, such as at $G = 0.5, 2$ and $4$, the vorticity generated in the $1^{st}$ half period and the $2^{nd}$ half period is quite close but not identical, especially on the inner surface of the cylinders. It is speculated that the slight difference in the vorticity generated in the two half periods contributes to the asymmetric flow features observed in the oscillatory direction, such as that shown in Figure 5-15. A noticeable difference between the two half periods on the wall vorticity is found at $G=1$, where cylinder 3 is in regime D and cylinder 4 is in regime A. Figure 5-30 (f & g) also illustrates the enstrophy integrated around the wall (inner side wall, outer side wall and the total, respectively) of two diagonal cylinders in an oscillation period, defined as,

$$\varepsilon = \int_{r \Omega} \left( \omega_z \right)^2 dV dt$$

(5-14)
where $\Omega$ represents the surface of the cylinder. On cylinders 2 and 4, less than 5% difference is found in the total enstrophy at $G=0.5$ and 1, and the difference is negligible at $G=2$ and 4. The total enstrophy on cylinder 2 and 4 are both smaller than that of the single cylinder for gap ratios investigated in the present study. The enstrophy on the inner side wall at $G=0.5$ and 1 is larger than that on a half of the single cylinder, and becomes almost identical to that on a half of the single cylinder at $G=2$ and 4. The enstrophy on the outer side wall is significantly smaller compared to that on a half of the single cylinder.

Figure 5-30 The change of wall vorticity with gap distances at $(Re, KC) = (160, 5)$. (a) ~ (e), $\omega_z$ on the wall; (f) and (g), comparison of $\varepsilon$: --, single total; ---, single $0 \sim \pi$; ---, single $\pi \sim 2\pi$; --, individual cylinder total; o, individual cylinder outer side wall; ●, individual cylinder inner side wall.

5-41
Figure 5-31 Comparison of drag and inertia coefficients for four circular cylinders in an in-line square arrangement with gap ratio of 1 and 4 with those of a single cylinder in the same flow condition.

5.4.2 Force coefficients

Hydrodynamic forces on the cylinder array are investigated and the total force coefficients are illustrated as a function of $KC$ and $Re$ in Figure 5-31. The total in-line forces $F_{y,T}$ are calculated by integrating the pressure and shear stress on the surfaces of all four cylinders. To facilitate the comparison with forces on a single cylinder, the total in-line force was divided by the number of cylinders before it was applied in Equation (2-7). Thus, the total drag and inertia force coefficients are defined as,

$$\frac{F_{y,T}}{4} = \frac{1}{2} \rho D C_{D,T} |U_y(t)U_y(t) + \rho \pi D^2 C_{M,T} \frac{dU_y(t)}{dt}$$

(5-15)
where $C_{D,T}$ and $C_{M,T}$ are the total drag and inertia force coefficients for the four-cylinder array, respectively. The total drag and inertia coefficients of the four cylinders are presented here for a medium gape ratio ($G=1$, Figure 5-31 c&d) and a large gap ratio ($G=4$, Figure 5-31 e&f), along with those of a single cylinder (Figure 5-31 a&b) under the same flow conditions.

For a single cylinder, the drag coefficient generally decreases with the increase of both Re and $KC$, with much steeper slopes at low $KC$ and Re than that at high $KC$ and Re, as indicated by the density of the contour lines. On the other hand, the inertia coefficient exhibits much more intricate features. For regimes A* and A (refer to Figure 5-7), $C_M$ is diagonally distributed in a trend of increase with the increase of $KC$ and a decrease trend with the increase of Re. However, for regime F, the force coefficient is in a pattern of reverse-diagonal distribution compared with those of regime A* and A. In the interface area of those two areas with diagonally distributed $C_M$ coefficient, regime D and E show a very unstable and irregular pattern in the $C_M$ distribution.

For the four cylinders at $G=1$, the total drag coefficient at small Re shares the same pattern of distribution as that of a single cylinder where the contour lines are diagonally distributed. Large difference is observed beyond Re $\geq 100$, especially at the high end of Re with $KC=5\sim6$, where an $H_2$ symmetric pattern is found in the flow patterns. Overall, $C_{D,T}$ is larger than that of a single cylinder at low $KC$ values but smaller at high $KC$ values. Generally, the drag coefficient is more evenly distributed. The large area of sparse contour lines at high Re for a single cylinder almost disappears in the four-cylinder case, except in a small region where $H_2$ symmetry occurs. The inertia coefficient of the four-cylinder array at $G=1$ across the map is similar to that of the single cylinder case at regimes A* & A, and the reversely-diagonal distribution of $C_M$ at large $KC$ values is not observed in the four-cylinder array.

The drag and inertia coefficients on the cylinder array at $G=4$ are very similar to that on a single cylinder, not only in the distribution, but also in the magnitude (only $C_M$ is slightly smaller). For instance, the drag shows similar contour lines with mild slopes at high Re while steep ones at low Re; in addition, the inertia coefficient’s distribution shows two zones of diagonally distributed contours and an irregular pattern at the regime where presumably regime D and E would occur if there were only one cylinder.

To summarize, the drag distribution at $G=1$ overall resembles a squeezed area at low
Chapter 5

KC of the single cylinder case, because the cylinder array behaves as a single structure at small gap ratios. This is consistent with the flow features observed in Figure 5-9. At large gap ratios, such as at $G=4$, drag distribution is almost equivalent to that of a single cylinder. Although the flow fields around four-cylinder array are much different from those around a single cylinder, the inertia and drag forces are less affected and are similar to those on a single cylinder.

Figure 5-32 Lift force coefficients ($C_{L}$) for four circular cylinders in an in-line square arrangement with gap ratio of 1 and 4.

The total lift coefficients ($C_{L,T}$) from two selected gap ratios are presented in Figure 5-32, which is defined as,

$$\frac{F_{Lx}}{4} = \frac{1}{2} C_{L,T} \rho DU_{m}^2$$ (5-16)

where $F_{Lx,T}$ is the total lift force integrated from the pressure and shear stress on the surfaces of all cylinders. It is evident that at zones of $K_x$ symmetry, the total lift coefficient for the cylinder array is zero, because that the lift force on the left pair and right pair are equal but with opposite signs. Whenever the flow regime breaks the $K_x$ symmetry, the asymmetric flow fields to the $x$-axis leads to a non-zero total lift coefficient. This is why the contour lines of lift coefficient distribution are very similar to the regime boundary lines as seen in Figure 5-9. It seems the lift coefficient experiences a dramatic rise with the increase of Re after the flow breaks the $K_x$ symmetry in both $H_2$ and $N$ regimes, indicating the increasingly chaotic flow fields.

5.5 Conclusions

The flow structures around four circular cylinders in an in-line square arrangement induced by sinusoidally oscillatory flow were numerically investigated. Two-
dimensional simulations were carried out at relatively low frequencies and amplitudes of oscillations within $KC \in [1, 12]$ and $Re \in [20, 200]$. The flow fields around the four-cylinder array are comprised of many combinations of the established flow regimes around a single cylinder identified by previous independent studies. These combinations show an even more captivating set of flow patterns around four circular cylinders than that around a single one. The flow features are classified into six types of reflection symmetry to the axis of flow oscillation, two types of spatio-temporal symmetry and a series of symmetric breaking flow patterns, which are mapped out in the Re-$KC$ plane. The reflection symmetry to the axis of oscillation dominates the maps of flow regimes, especially at low gap distances, due to the stabilization effect induced by the jet-like flow through the gap formed by cylinder columns that are parallel to the flow oscillation direction. In general, at small gap distances, the four structures behave as a single porous body and therefore, the flow fields resemble those around a single cylinder; while with the increase of gap distance, the individual behavior of flow around each cylinder in the array starts to influence the overall flow patterns and thus, the flow shows a variety of symmetry patterns as a result of vortex interactions from each cylinder, but the symmetric patterns are also found to be prone to asymmetry. It is also found that three-dimensionality of the flow does not appear to significantly influence the cross-sectional flow fields or the onset of symmetry breaking regime. The drag force coefficient of the four cylinders as a whole at low gap distance shows a trend of variation similar to a squeezed area at low $KC$ of the single cylinder case, but gradually changes to the same pattern with the increase of the distance among the cylinders. The drag force seems to be less influenced compared to the flow fields at large gap distances. The lift coefficient is zero for the cylinder array at the reflection symmetry regimes, and experiences a dramatic increase with $KC$ and Re once the symmetry flow is broken.

**Acknowledgement**

This work was supported by Australian Research Council Discovery Grant (Project ID: DP110105171) and by iVEC through the use of advanced computing resources (Epic and Magnus supercomputers). F. T. would like to acknowledge the support of the Australian Government and the University of Western Australia by providing SIRF, UIS and Completion scholarships for a doctoral degree.

**REFERENCES**


2317-2338.


Chapter 6

Concluding Remarks

6.1 Main findings

This thesis has focused on the flow features around multiple-circular cylinders and their impacts on the forces imposed on the structures. The research is particularly driven by the common occurrence of cylindrical structures in groups in engineering applications including offshore structures and high buildings; and by the rich physics displayed by the flow, such as how the vortices are developed and shed under the disturbance from neighbor counterparts. Both two-dimensional and three-dimensional simulations were carried out, depending on the Reynolds number of the fluids, after sufficient numerical validations. The major contributions and findings made in this research are summarized below.

I. Steady uniform flow around two staggered cylinders was numerically modeled at a low subcritical Reynolds number of $10^3$. The flow through and around the two-cylinder system were analyzed for pitch ratios ($P/D$) of 1.5, 2, 3 and 4 with the flow incident angle ($\alpha$) varying between $0^\circ$ and $90^\circ$.

1. The pressure distribution on each of the two cylinders changes significantly with the change of pitch ratio and alignment angle, especially when $P/D \leq 3$ and $\alpha \leq 60^\circ$. The movements of stagnation points and variations in the base pressure and stagnation pressure are the manifestations of the vigorous interference in the flow field under the presence of another cylinder. The change in pressure
distribution leads to a variety of changes in the forces on both cylinders. Negative drag on the downstream cylinder is found for cases with small alignment angles at medium pitch ratios ($\alpha \leq 5^\circ$ and $P/D \leq 3$). At medium alignment angles ($5^\circ \leq \alpha \leq 30^\circ$), the drag force on the downstream cylinder is less than that of an isolated cylinder under the same flow condition and the averaged lift coefficient is characterized by the attractive force. At large alignment angles ($\alpha \geq 30^\circ$), the drag forces on the two cylinders are less than 15% different from that of a single cylinder, while repulsive force is found in the lift at small pitch ratios ($P/D < 3$) but fades out with the increase of pitch ratio.

2. Flow regimes are identified through flow visualizations and FFT analysis of lift forces and velocity signals sampled from the wake of the cylinders. Following a notation from a previous study on flow classifications, four flow regimes are identified and are mapped on the $P/D$-$\alpha$ plane. Regime S-I is characterized by a single wake and one $St$ for both cylinders, while regime S-II by a single merged wake and two distinct $St$ for the cylinders. Vortices are shed from both cylinders at different frequencies and maintain to be at different frequencies in the wake regime T-I. Regime T-II was only found $P/D = 3$ and 4 and at $\alpha = 90^\circ$, with two wakes but the same vortex shedding frequency. It was observed that the wake interference at small pitch ratios in regimes S-I and S-II weakens the three-dimensionality around two staggered cylinders, which applies not only to the gap region between the two cylinders, but also to the shared wake of both cylinders. This was qualitatively shown by the 3-D flow fields and quantitatively demonstrated by the enstrophy and spanwise pressure fluctuations.

3. Overall the 2-D model is not able to predict flow structures correctly at this Reynolds number because (1) it predicts the inception of vortex shedding from the upstream cylinder at a smaller pitch ratio than its actual pitch ratio and (2) it predicts more energetic wakes than the 3-D model. Therefore it is not appropriate to use 2-D simulations to classify the flow fields around two staggered cylinders. The 2-D simulations did yield fairly comparable drag coefficients when the wake and shear layer interactions are less active in regimes T-I and T-II. Reasonable predictions were also found on the movement of stagnation points, on mean lift coefficients and on the total enstrophy calculations for the same cases. However, in regime S-I and S-II, the 2-D predictions were less reliable.
II. Steady uniform flow around a four cylinder array in a square arrangement was simulated numerically at relatively small Reynolds numbers ranging from 50 to 300 with an interval of 10 and pitch ratios from 1.7 to 4.5. Wake characteristics behind cylinders and hydrodynamic forces on the cylinders are investigated by carrying out a series of numerical tests. A total of seven flow regimes are identified and are mapped onto the Re–$P^*$ plane. Each of the seven flow regimes, which are referred to as Pattern A to Pattern G for the convenience of discussion, has distinguished flow features, resulting from the interactions among shear layers around the cylinders, Kármán vortex streets behind downstream cylinders and vortices shed around cylinders. The detailed flow features and force characteristics on the cylinders are summarized below.

4. No vortex shedding is found in pattern A at small Reynolds numbers. Pattern B and pattern C are featured with a single wake at spacing ratios smaller than 3.3 and Reynolds numbers greater than approximately 70. The difference between pattern B and pattern C is that vortices are found to shed from the inner side shear layers in pattern C. Two parallel streets of Kármán vortex, which are symmetrically distributed along the longitude centreline, dominate the wake of flow pattern D. In the in-phase vortex shedding pattern E, one (pattern E1) and three (pattern E2) streets of Kármán vortex synchronization are detected. Biased vortex shedding, one of the main flow features for two cylinders in a side-by-side configuration, is also present in flow around the four circular array in pattern F, and the switchover from one deflected side to the other is also observed. In pattern G, located at large Re and large spacing ratios, the wake is featured with vortex co-shedding from both up- and down-stream cylinders, which are symmetrically distributed.

5. Vortex shedding from the upstream cylinders is observed in patterns E, F and G at large spacing ratios when the Reynolds number is greater than about 80. The onset of vortex shedding from upstream cylinders shares similar features with those of the in-phase mode and is referred to as “in-phase escape” (except the largest spacing ratio considered), where the wake flow immediately behind the upstream cylinders moves to the same direction in order to pass over the downstream cylinders when the onset of vortex shedding takes place.

6. Hydrodynamic forces on the downstream cylinders are found to be relatively small in flow pattern A to pattern D where vortex shedding from upstream
cylinders does not take place. The hydrodynamic forces on the downstream cylinders jump to large values at pattern E due to vortex shedding from upstream cylinders. The root-mean-square lift force on downstream cylinders increases consistently with the increases of Re and spacing ratio. The drag forces on the rear cylinders appear to decrease slightly at large spacing ratios \((\geq 3.0)\) and at large Re \((\geq 200)\). The total lift coefficient on the four-cylinder array as a whole is close to zero at patterns D and G due to the anti-phase pattern of the flow field, while it is much larger at patterns E and F where the in-phase or biased vortex shedding dominates the wake.

III. Uniform flow past four circular cylinders in an in-line square configuration with a constant spacing ratio of 2 was studied numerically by directly solving the three-dimensional Navier-Stokes equations. The focus was to examine the effects of Reynolds number on flow regimes and hydrodynamic forces on the cylinders. A range of Reynolds number from 100 to 500 was considered.

7. The flow can be classified into four regimes based on Reynolds number. The flow in regime 1 \((100 \leq \text{Re} \leq 220)\) is characterized by the occurrence of the weak and inclined spanwise vortices. The streamwise vortices are also very weak. The four cylinders largely behave as a single structure; only one vortex street is formed behind the four cylinder array. The flow in regime 2 \((240 \leq \text{Re} \leq 300)\) is characterized by the regular wavy spanwise vortices and rib-shaped streamwise vortices. The wavelength of the spanwise vortices is about 1.2. Two distinct vortex streets are observed after the four-cylinder array. The flow in regime 3 \((320 \leq \text{Re} \leq 380)\) is characterized by strong vortex dislocations in the wake of the cylinders. The two Kármán vortex streets in regime 3 are mostly synchronized in an in-phase mode, intermittently switching to an anti-phase synchronization. The flow regime 4 \((400 \leq \text{Re} \leq 500)\) is characterized by the ceasing of vortex dislocations and strong streamwise vortices. There are four interacting wakes in regime 4. The strong interactions between the four wakes lead to many fine-scale vortices in the wake behind the four-cylinder array. The flow between the upstream and the downstream cylinders is 2-D in regimes 1, 2 and 3 and becomes 3-D in regime 4.

8. When the flow transits from one regime to another, significant changes occur in the force coefficients. The mean drag coefficients on the two downstream
cylinders are much smaller than those on the upstream ones because the former are in the wake of the upstream cylinders. The RMS lift coefficients of the two downstream cylinders are relatively small in regimes 1 and 3 because of the inclination and the weakness of vortices in regime 1 and the vortex dislocation in Regime 3. The RMS lift coefficients of the two downstream cylinders are relatively large in Regimes 2 and 4 because the spanwise vortices in these two flow regimes are parallel to the cylinders and there are no vortex dislocations. The peak frequencies of the lift force and the drag force are found to be almost the same for any single cylinder in the array. It is also found that the Strouhal number values for individual cylinder in the array are within 2% difference over the Reynolds number range considered, due to the synchronized vortex shedding from the cylinders.

IV. The flow structures around four circular cylinders in an in-line square arrangement induced by sinusoidally oscillating flow were numerically investigated. Two-dimensional simulations were carried out at relatively low frequencies and amplitudes of oscillations within $KC \in [1, 12]$ and $Re \in [20, 200]$, the range of which investigated by the 2-D numerical model is supported by published works and is carefully validated in the present study.

9. The flow fields around the four-cylinder array are comprised of many combinations of the established flow regimes around a single cylinder by previous studies. These combinations show an even more captivating set of flow patterns around four-circular cylinders than that around a single one. The flow features are classified into six types of reflection symmetry to the axis of flow oscillation, two types of spatio-temporal symmetry and a series of symmetric breaking flow patterns and mapped out in the Re-$KC$ space at four gap distances. The reflection symmetry to the axis of oscillation dominates the maps of flow regimes, especially at low gap distances, due to the stable effect induced by the jet-like flow through the cylinder array. In general, at small gap distances, the four structures behave as a single porous body and therefore, the flow fields resemble to those around a single cylinder; while with the increase of gap distance, the individual flow behavior around each cylinder in the array starts to influence the overall flow patterns and thus, the flow field shows a variety of
symmetry and asymmetry patterns because of vortex and shear layer interactions.

10. The drag force coefficient of the four cylinders as a whole at low gap distance shows a trend of variation similar to a squeezed area at low $KC$ of the single cylinder case, but gradually changes to the same pattern with the increase of the distance among the cylinders. The drag force seems to be less influenced by the appearance of other cylinders compared to the flow fields at large gap distances. The lift force coefficient is zero for the cylinder array at the reflection symmetry regimes, and experiences a dramatic increase with $KC$ and Re once the symmetry state is broken.

### 6.2 Recommendations for future work

The modelling on the flow features around multiple-circular cylinders has carried out in this research. Further investigations that can be made in the future study are outlined below:

1. Oscillatory flow around a single cylinder in regime C is not well documented in the reviewed literature. Regime C only occupies a small regime in the space of Re-$KC$, yet it displays a quasi-periodic state that no any other regimes have. The mechanisms of how this flow regime generates and disappears have not been well studied.

2. This work provides a way, through two-dimensional (2-D) direct numerical simulations, to differentiate flow regimes around four cylinders in both steady and oscillating flow, and the results are useful for the understanding of the physics of the flow around multiple cylinders. However, it must be admitted that experimental studies & expensive 3-D studies seem to be inevitable in order to better understand the flow structures at the Reynolds number that is known to harbour 3-D flows.
Appendix A

The scalability of the CFD simulations on two supercomputers for flow around a cylinder

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Abstract: This paper presents a detailed study on the scalability of OpenFOAM, a widely used open-source code for computational fluid dynamics (CFD), on two parallel clusters. The objective of the study is to provide a guide on optimizing CPU cores when carrying out simulations on parallel clusters with the open-source framework. The study is based on flow around a circular cylinder at early turbulent regime ($Re = 2000$). The computational cell number ranges from 15,000 to 18 million; while the applied number of MPI processors from 1 to 2048. The results suggest that OpenFOAM scales very well on the two clusters. The workload for one processor is suggested based on the cases studied, and the influence of solvers and supercomputers are summarized.

A.1 Introductions

OpenFOAM (Open Field Operation and Manipulation) has been gaining increasing popularity in CFD community, due to its well designed characteristics, such as free and open-source codes, highly modular programming and reliable and efficient calculations.

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The parallel computing has become essential in CFD modeling, with many industrial problems requiring large scale simulations for rather complex geometries. The parallelization of OpenFOAM is performed by means of MPI (Message Passing Interface), which are widely used in different application areas.

Efforts have been made to understand the scalability of OpenFOAM on supercomputers. Most of the scalability studies so far have been conducted based on the laminar lid-drive cavity flow where a steady state solution exists. The study was firstly scaled with 256 MPI tasks (Calegari et al., 2009), later up to 4096 MPI tasks for 8 million cells (Pringle, 2010), and more recently with 4608 MPI tasks for 27 million cells (NTNU_HPC_Group, 2012). So far, the scalability of OpenFOAM has been tested on several highly ranked supercomputers around the world, for instance, on TGCC Curie (Culpo, 2011) and the HPC system Vilje (NTNU_HPC_Group, 2012). Several solvers for laminar flow, turbulent flow and multiphase flow that are available in OpenFOAM were studied (NTNU_HPC_Group, 2012, Pringle, 2010, Culpo, 2011, Rivera and Furlinger, 2011). The previous studies found that OpenFOAM is generally of linear or superlinear scalability in parallel computing, but the limit of the scalability changes from case to case, largely depending on how many cells in one run, the classes of parallel computers used and the choice of MPI software. Some studies (2011, NTNU_HPC_Group, 2012) have also suggested that different linear equation solvers is an above all significant factor to the performance of supercomputing and can influence both calculation speed and efficiency.

Unlike previous scalability studies, the present study is carried out based on uniform flow around a circular cylinder at an early turbulent regime ($Re = 2000$), a popular flow problem where steady state solution does not exist. To reach to the equilibrium state of vortex shedding, a large number of time steps are needed in simulations of this nature which is profoundly different from simulating problems with steady-state solutions. In addition, the scalability of laminar and turbulent flow solvers is also investigated. The tests are carried out on two iVEC facilities located in Western Australia, named Epic and Magnus. Epic is a HP commodity cluster which runs CentOS Linux and Magnus is a Cray XC30 supercomputer (Ryan and Hartman-Baker, 2013).

A.2 Methodology

This section describes the benchmarking cases, the solvers, the supercomputers and
the evaluation method used in this study.

\subsection*{A.2.1 Benchmarking cases}

Uniform flow around a circular cylinder at early turbulent regime is employed as the benchmarking case for the current scalability study. Uniform flow around a circular cylinder is a classical fluid mechanics problem which has been widely studied both numerically and experimentally due to its wide applications in engineering and rich fluid mechanics. It is well known for the occurrence of von Karman vortices when Reynolds number is greater a critical value about 40. The Reynolds number is defined as \( Re = \frac{UD}{\nu} = 2000 \), where \( U \) is velocity of the cylinder with a diameter \( D \) relative to the fluid, \( \nu \) is the kinematic viscosity. A \( Re \) of 2000 is considered in the present study, where it is known that the cylinder wake has developed considerable three-dimensionality, but can still be captured by model computational capacity with Direct Numerical Simulation (DNS).

(a) \hspace{1cm} (b)

![Image](image1)

Figure A-1 A schematic representation of the circular cylinder (a) and the computational mesh around the cylinder (b).

A rectangular domain as shown in Figure A-1 is used in the calculation. A two-dimensional (2D) structured mesh is generated with 15,000 quadrilateral elements in the \( x \)-\( y \) plan, with finer mesh around the surface of the cylinder. The mesh is then extruded in the \( z \)-direction in equidistance in the \( z \)-direction of certain layers to increase the number of cells required for the scalability tests. Table A-1 lists the cases in this study. In Table A-1 the case name is given indicating the number of total cells used in the test. For example, case 0.09M indicates that the total cells in this case is 0.09 million, which is obtained by extruding the 2D mesh with 6 layers in the \( z \)-direction, while the 18 million cells in case 18M are contains 1200 layers in the \( z \)-direction.
A.2.1 icoFoam and pisoFoam

Two transient, incompressible solvers of icoFoam and pisoFoam in OpenFOAM 2.x are considered in this study. The icoFoam is a solver for laminar flow, while pisoFoam, users have the option to turn on turbulence models. The standard $k$-$\varepsilon$ turbulence model was chosen for turbulent flows in this study.

In both solvers, a uniform velocity $U$ in the $x$-direction and zero velocity in the $y$- and $z$-directions are specified at the inlet boundary. The pressure outlet condition is applied at the outlet boundary. The cylinder surface is set as no-slip wall, while the top and bottom boundaries are set as symmetric planes, where normal gradient is zero. Linear interpolation is used for the volume flux calculations and the equations are advanced in time using an Euler scheme. These solvers employ a PISO (Pressure Implicit with Splitting of Operators) velocity-pressure coupling approach to solve the Navier-Stokes equations. A GAMG (Geometric Algebraic Multi Grid) iterative method is used for the solution of the pressure equation with two steps of corrections, while the momentum and transport equations are solved using a preconditioned bi-conjugate gradient (PBiCG). In all test cases, a fixed time step is specified to keep CFL condition number less than 0.4. The icoFoam is investigated for all cases listed in Table A-1 while pisoFoam’s scalability performance is only considered for case 9M.

<table>
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</tr>
<tr>
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<td>9</td>
<td>icoFoam/pisoFoam</td>
</tr>
<tr>
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<td>3</td>
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<td>1.5</td>
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<td>0.5</td>
<td>icoFoam</td>
</tr>
<tr>
<td>0.09M</td>
<td>0.09</td>
<td>icoFoam</td>
</tr>
<tr>
<td>0.015M</td>
<td>0.015</td>
<td>icoFoam</td>
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</tbody>
</table>

Table A-1 Benchmarking cases for the present study

A.2.2 Epic and Magnus

The tests are carried on two iVEC facilities located in Western Australia, named Epic and Magnus. Epic is a HP commodity cluster which runs CentOS Linux and Magnus is a Cray XC30 supercomputer (Ryan and Hartman-Baker, 2013). The details of these two Linux clusters are listed in Table A-2. As a whole, Epic is quicker in core clock rate and considerably larger than Magnus in terms of number CPU cores; however, Magnus outperforms Epic in RAM/node, cores/node and the total storage. OpenFOAM is installed on Epic with gcc compiler and the MPI implementation is performed through
OpenMPI, while on Magnus, the standard MPI linked to OpenFOAM is MPICH2.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Epic</th>
<th>Magnus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>800</td>
<td>208</td>
</tr>
<tr>
<td>CPU Cores</td>
<td>9600</td>
<td>3328</td>
</tr>
<tr>
<td>Cores per node</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>RAM per node(GB)</td>
<td>24</td>
<td>64</td>
</tr>
<tr>
<td>Storage (PB)</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>GH/s/core</td>
<td>2.97</td>
<td>2.60</td>
</tr>
<tr>
<td>Interconnect(G/s)</td>
<td>QDR</td>
<td>Infiniband(≈ 23–40 G/s*) Cray Aries (72 G/s)</td>
</tr>
<tr>
<td>OpenFOAM platform</td>
<td>gcc / OpenMPI</td>
<td>cc / MPICH2</td>
</tr>
</tbody>
</table>

Table A-2 Details of the two supercomputers

* value estimated based on training materials from iVEC

### A.2.3 Timing and performances

The computing performances of two parallel clusters are evaluated by comparing the execution time required for a single time step in a calculation. For each case listed in Table A-1, twelve tests are performed to investigate the computation speed, with \( n = 2^i \) processors respectively, where \( i \) ranges from 0 to 11. As a results, the number of MPI processors used in the tests ranges from 1 to 2048. In each test, a wall time (as in a clock on the wall) of six hours is executed to reduce the effect of startup overhead (where the speed of computing is very low at the start of a calculation), except for the test cases with 2048 processors, for which a wall time of one hour is applied to save computing resources. Then the execution wall time for a single step is estimated by how many time steps calculated in these six hours (or one hour for 2048-processor cases), i.e., the startup overhead is not excluded from the analysis.

The parallel computing performance is also estimated by the speedup, which is defined as the ratio of execution time between a sequential algorithm and a corresponding parallel algorithm. The speedup with a number of \( n \) processors is defined as,

\[
S_n = \frac{T_1}{T_n}
\]  

(A-1)

where \( T_1 \) is the execution time with one processor, \( T_n \) is the execution time with \( n \) processors or MPI tasks. A speedup of \( n \) is often referred as ideal or linear speedup when \( n \) processors are used. Normally \( S_n \) is smaller than \( n \), although a super-linear speedup where \( S_n > n \), has also been observed in parallel computing(NTNU_HPC_Group, 2012).
Two kinds of output data, except the standard output, are written to the hard drives during the calculation, in order to include the input/output (I/O) performance of the clusters. One kind of the output is the hydrodynamic forces on the cylinder every 20 time steps, through the standard forces library (libforces.so) of OpenFOAM. The other kind of output is the simulation snapshots of velocity and pressure fields every 250 time steps.

Figure A-2 The execution wall time for a single time step of parallel computing with OpenFOAM on Epic as a function of MPI processors

Figure A-3 The execution wall time for a single time step of parallel computing with OpenFOAM on Magnus as a function of MPI processors
A.3 Test Results

A.3.1 Wall time

The variations of averaged wall clock time with number of MPI processors for a single time step are shown in Figure A-2 and Figure A-3 for Epic and Magnus, respectively. It should be noted that both Figure A-2 and Figure A-3 are plotted in log scales. It is observed for all the cases on both clusters that the required averaged wall clock time for a single time step calculation decreases with the increase of number of processors until a critical number of processors beyond which the required averaged wall clock time for a single time step calculation starts to decrease. For instance, the minimum wall time value for 18M is about 1.3 seconds (s) at 1024 processors on Epic and employing more processors does not shorten the computing time. The curves representing different number of computational cells are almost parallel before the minimum execution times are reached. As expected, the averaged wall clock time for a single time step calculation increases with the total number of computational cells in the domain. The parts of the curves after the minimum execution times are reached are not shown in Figure A-2 and Figure A-3 to keep the figures less crowded. Based on the results presented in Figure A-2 and Figure A-3, one can optimize the use of processes and estimate the approximate run time required for a given work and number of processors available.

<table>
<thead>
<tr>
<th>MPI processors</th>
<th>Epic (s)</th>
<th>Magnus (s)</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>139.35</td>
<td>113.10</td>
<td>+23%</td>
</tr>
<tr>
<td>2</td>
<td>65.65</td>
<td>54.55</td>
<td>+20%</td>
</tr>
<tr>
<td>4</td>
<td>42.60</td>
<td>31.26</td>
<td>+36%</td>
</tr>
<tr>
<td>8</td>
<td>30.81</td>
<td>26.37</td>
<td>+17%</td>
</tr>
<tr>
<td>16</td>
<td>17.36</td>
<td>11.95</td>
<td>+45%</td>
</tr>
<tr>
<td>32</td>
<td>11.25</td>
<td>5.70</td>
<td>+97%</td>
</tr>
<tr>
<td>64</td>
<td>5.70</td>
<td>2.75</td>
<td>+107%</td>
</tr>
<tr>
<td>128</td>
<td>2.57</td>
<td>1.19</td>
<td>+116%</td>
</tr>
<tr>
<td>256</td>
<td>1.32</td>
<td>0.66</td>
<td>+101%</td>
</tr>
<tr>
<td>512</td>
<td>0.73</td>
<td>0.44</td>
<td>+64%</td>
</tr>
<tr>
<td>1024</td>
<td>0.54</td>
<td>0.34</td>
<td>+58%</td>
</tr>
<tr>
<td>2048</td>
<td>0.70</td>
<td>0.43</td>
<td>+61%</td>
</tr>
</tbody>
</table>

Table A-3 Comparison of execution wall time for a single time step between Epic and Magnus for case of 9M at different MPI processors

It appears that OpenFOAM performs better in parallel computing on Magnus than on Epic because the averaged wall clock time for a single time step calculation on the Magnus is consistently smaller than that of Epic for all the cases studied. The averaged wall clock time for a single time step calculation for case 9M on the two clusters is
compared in Table A-3. The results show that computing on Magnus is significantly quicker than on Epic. Epic is surprisingly about 100% slower than Magnus when 32–256 processors are used, about 60% slower when 512 or more MPI processors are used. The differences in in hard wares and MPI implementations may contribute to the scalability performance of OpenFOAM on two clusters.

![Figure A-4 The speedup performance of OpenFOAM on Epic for parallel computing](image)

![Figure A-5 The speedup performance of OpenFOAM on Magnus for parallel computing](image)

**A.3.2 Speedup**

The speedups of the parallel computing on Epic and Magnus are illustrated in Figure A-4 and Figure A-5, respectively. Compared with the ideal speedup, OpenFOAM’s
performance on the two clusters exhibits super-liner speedup only at the small number of parallel jobs (as seen at 2 MPI processors), and for most MPI tasks the speedup lines are lower than the ideal speedup and linearly increase up to a certain number of processors, followed by a sudden drop. For both computers, the speedup of OpenFOAM is generally better for cases with more cells than cases with fewer cells. Generally, the speedup lines are closer to the ideal speedup on Magnus, suggesting better scalability on Magnus, which is consistent with the data presented in Table A-3.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Optimum no. processors</th>
<th>Workload/processor</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03M</td>
<td>8</td>
<td>3,750</td>
</tr>
<tr>
<td>0.09M</td>
<td>16</td>
<td>5,625</td>
</tr>
<tr>
<td>0.5M</td>
<td>128</td>
<td>3,906</td>
</tr>
<tr>
<td>1.5M</td>
<td>256</td>
<td>5,859</td>
</tr>
<tr>
<td>3M</td>
<td>512</td>
<td>5,859</td>
</tr>
<tr>
<td>9M</td>
<td>1024</td>
<td>8,789</td>
</tr>
<tr>
<td>18M</td>
<td>1024</td>
<td>17,578</td>
</tr>
</tbody>
</table>

Table A-4 Summerization of the workload for a single processor for cases with different cells

The optimum number of processors beyond which the speedup does not increase with the increase of number processors varies from case to case as shown in Figure A-4 and Figure A-5. It is observed that the optimum number of processors generally increases with the number of computational cells. Table A-4 lists the optimum number of processors for all the cases tested in this study and the work load per processor. Coincidently the optimum numbers of processors are identical for two clusters. It appeared that the work load per processor at the optimum number of processors is generally low, smaller than 9,000 cells per processor for cases with less than 9 million cells. More workload per processor should be considered for large scale simulations, as illustrated by the 18M case in Table A-4.

A.3.3 Influence of solvers

Figure A-6 compares the wall time and the speedup between two solvers, icoFoam and pisoFoam, for case 9M. It is observed that pisoFoam is generally slower than icoFoam. This is because pisoFoam solve two more equations (k and ε equations) than icoFoam. With 1024 MPI processors, the execution wall time of a single time step for icoFoam and pisoFoam are 0.34s and 0.64s respectively. The trend of the speedup for both solvers is similar, but it is interesting to see pisoFoam scales up to 2048 processors for the case, instead of 1024 processors.
Appendix A

The University of Western Australia

A-10

A.4 Conclusions

The main objective of this work is to investigate the scalability features of OpenFOAM on two supercomputers. The results show that OpenFOAM scales very well on both computers but better performance on the Cray machine. It is suggested that 3500~6000 cells per processor is used for optimum speedups for simulation with less than 3 million cells, while more workload per processor should be considered for large scale simulations. The solver of pisoFoam computes generally slower than icoFoam, but the former scales further with the number of processors than the later.

Acknowledgments

This work was supported by Australian Research Council Discovery Grant (Project ID: DP110105171) and by iVEC through the use of advanced computing resources located at iVEC@Murdoch. F.T. would like to acknowledge the support of the Australian Government and the University of Western Australia by providing SIRF and UIS scholarships.

A.5 References


Appendix A


